Evaluating the Validity of Turbulent Kinetic Energy Dissipation Rate derived from Insitu 1D velocity fluctuation measurements

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Background

- Turbulent kinetic energy dissipation rate (***ε***) is a fundamental parameter characterizing the structure and intensity of turbulent flows.

- Measurements of Instantaneous turbulent kinetic dissipation rate requires the complete knowledge of 3D velocity field at high spatial and temporal resolution.

- Such measurements require very high sampling cadence (~10s KHz) and low-noise ($<10^{-8}$ m^2s^{-2}/Hz) (or very high SNR).

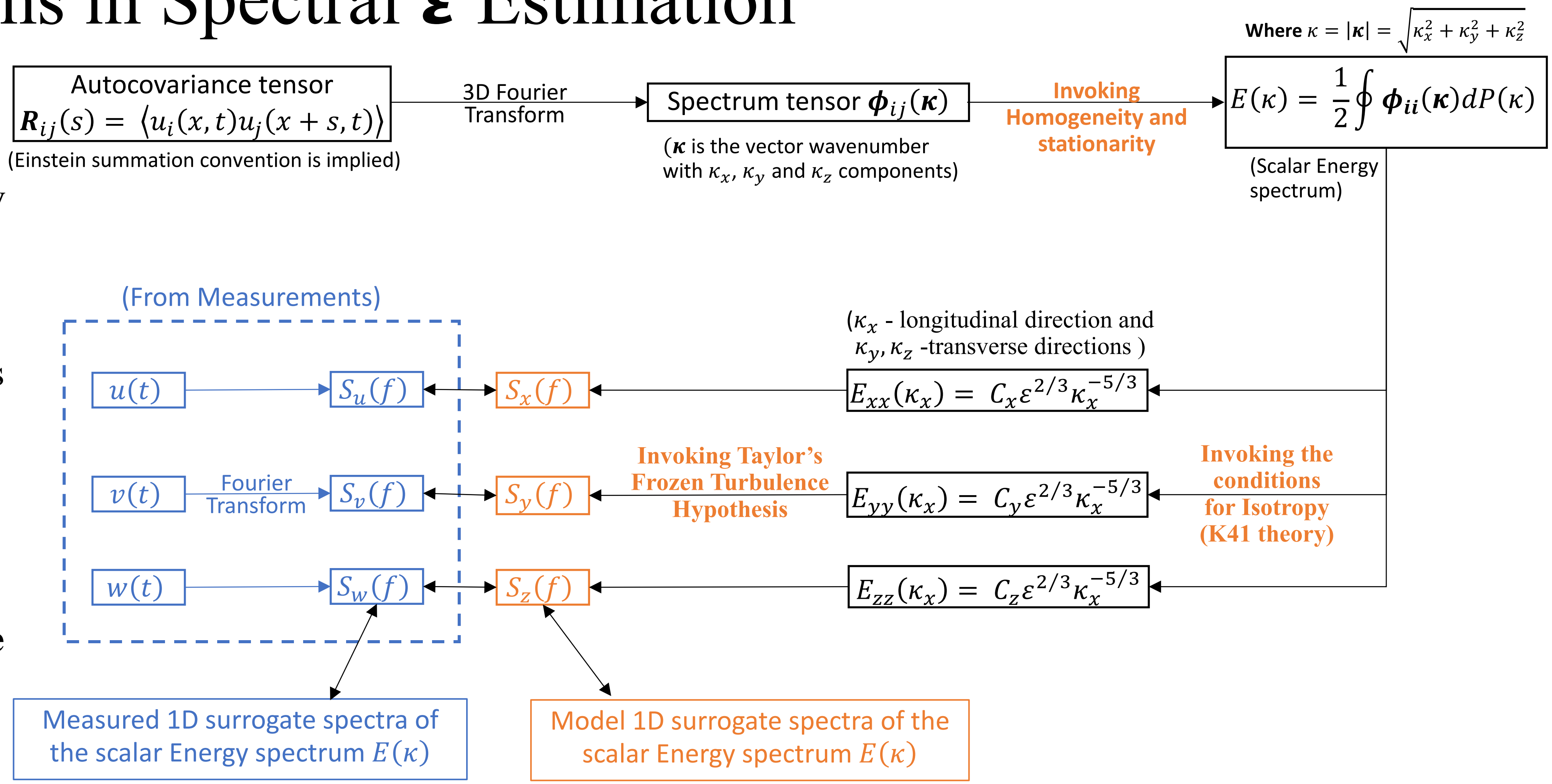
- Insitu measurements of atmospheric turbulence made using ground based and airborne instruments provide coarsely sampled time-series data of velocity components.

- Typically, only spatially and temporally averaged estimates of ***ε*** are derived from 1D insitu measurements.

- The 1D measurement data is subject to either a model **second-order structure function fit** or a **model turbulence energy spectrum fit** (in the inertial and/or viscous subrange) based on the hypotheses of Kolmogorov's turbulence theory.

Assumptions in Spectral ε Estimation

- These assumptions hold true for stationary HIT.
- Insitu observations intercept various states of atmospheric turbulence that likely do not conform to stationary HIT.
- **Necessary to evaluate the applicability of the spectral estimation method to atmospheric turbulence measurements.**



Synthetic Observations using DNS

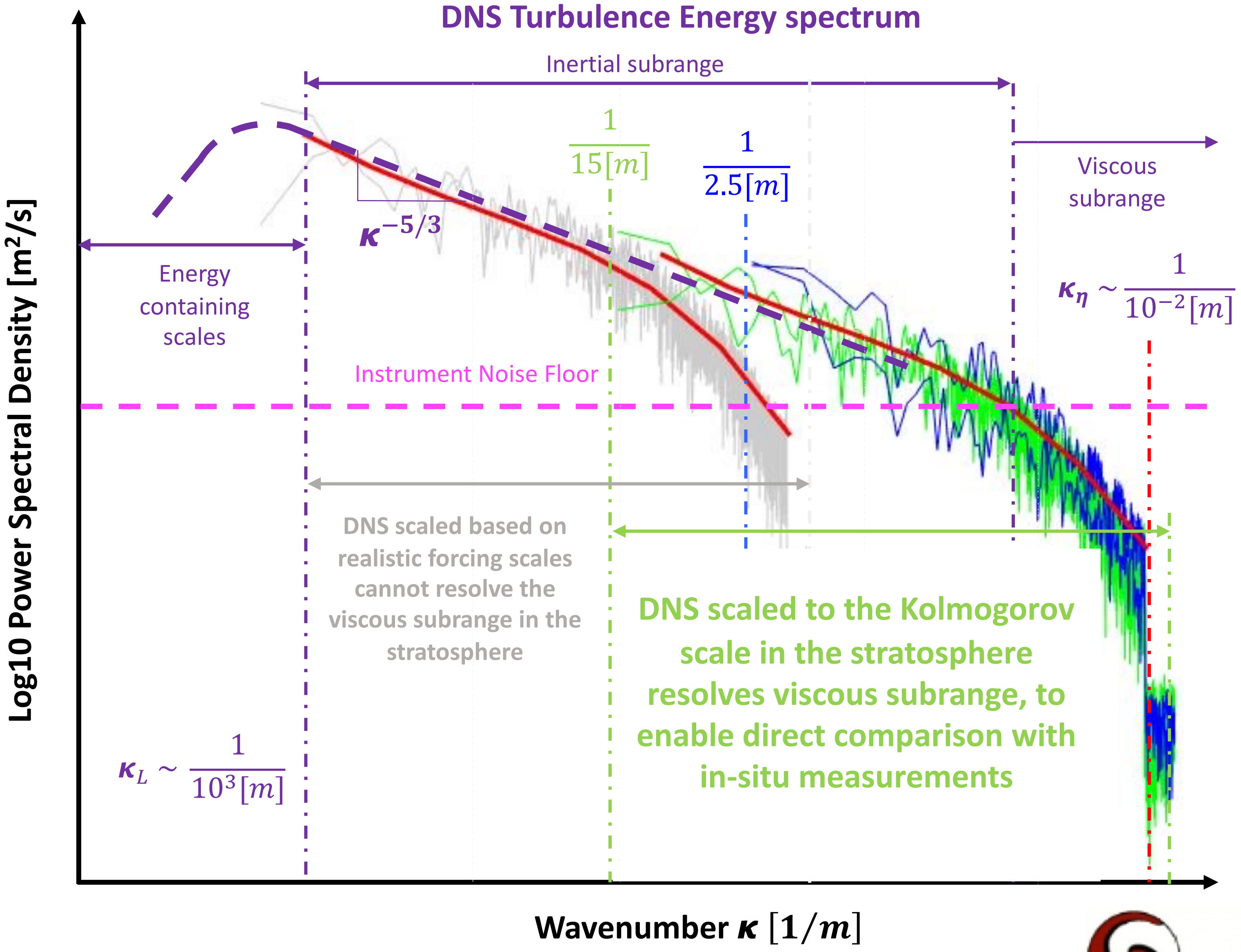
- Provides a “truth” data set for conducting synthetic observations to validate the spectral methods used for in-situ measurements of turbulence intensity (TKE dissipation rate).

- Allows assumptions commonly made in the spectral methods to be tested directly.

- Nontraditional DNS scaling used to match Kolmogorov scale and dissipation rate seen in the stratosphere.

- Well-validated spectral shape enables cm-scale turbulence intensity to be indirectly measured/predicted.

- Similar synthetic observations for deriving $\boldsymbol{\varepsilon}$ from radar observations (1-3 [km]) (using LES), sounding rockets (80-90 [km]) (using DNS), and flying hotwire measurements in the wind tunnel (using DNS) are described in **Lundquist et al 2020, Strelnikov et al 2021, and Schroder et al 2023.**



DNS Setup and Metrics

Model Setup

- Turbulence Reynolds Number $Re \sim \left(\frac{L}{\boldsymbol{\eta}}\right)^{4/3} \sim \mathbf{12,000}$

$\boldsymbol{\eta}$ – Kolmogorov length scale, **L -** DNS box size

- *Forcing:* Energy injected at wavenumbers $\left(\frac{2\boldsymbol{\pi}}{L}\right), \left(\frac{4\boldsymbol{\pi}}{L}\right), \left(\frac{6\boldsymbol{\pi}}{L}\right), \left(\frac{8\boldsymbol{\pi}}{L}\right)$

Scaling

- Non-dimensional DNS results are scaled using HYFLITS measurements of $\boldsymbol{\varepsilon}$ (TKE dissipation rate) and $\boldsymbol{\nu}$ (Kinematic viscosity)

  $\boldsymbol{\varepsilon}_M = 2.9e^{-3} \left[{}^{m^3}/{}_{s^2}\right]$ and $\boldsymbol{\nu}_M = 4e^{-4} \left[{}^{m^2}/{}_{s}\right]$ sampled at 30 km

- **Kolmogorov length** and **time** scales are constructed using measured $\boldsymbol{\varepsilon}$, $\boldsymbol{\nu}$

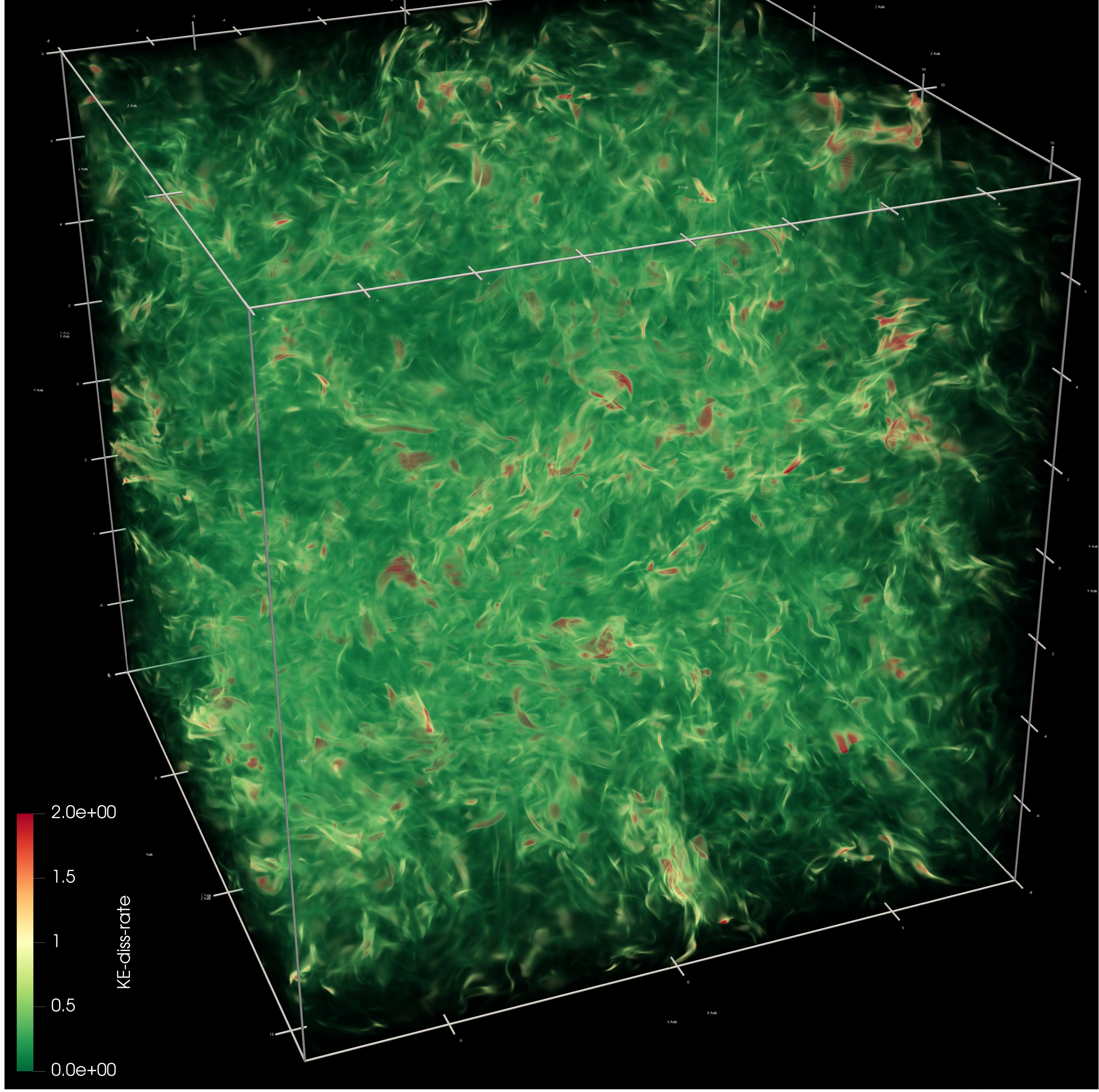
$$\boldsymbol{\eta}_M = \left(\frac{\boldsymbol{\nu}^3}{\boldsymbol{\varepsilon}}\right)^{1/4} \quad \text{and} \quad \boldsymbol{\tau}_M = \left(\frac{\boldsymbol{\nu}}{\boldsymbol{\varepsilon}}\right)^{1/2}$$

- $\widetilde{\boldsymbol{\varepsilon}}_i = \widehat{\boldsymbol{\varepsilon}}_i\, C_{\boldsymbol{\varepsilon}} \qquad \widetilde{u}_i = \widehat{u}_i\, C_u \qquad$ i = 1, 2, 3, … N^3

[N = number of grid points in X/Y/Z direction; **^ Unscaled DNS; ~ scaled DNS**]

- $C_{\boldsymbol{\varepsilon}} = \frac{\boldsymbol{\varepsilon}_M}{\langle \widehat{\boldsymbol{\varepsilon}}_i \rangle} \quad C_{\boldsymbol{\eta}} = \frac{\boldsymbol{\eta}_M}{\langle \boldsymbol{\eta}_{DNS} \rangle}$ and $C_{\boldsymbol{\tau}} = \frac{\boldsymbol{\tau}_M}{\boldsymbol{\tau}_{DNS}}$ provide $C_u = \frac{C_{\boldsymbol{\eta}}}{C_{\boldsymbol{\tau}}}$
- Scaling results in **box size of 15 [m]** (in X, Y, and Z); and **grid resolution of $\boldsymbol{\delta}$ = 0.02 [m]**

3D volume visualization of TKE dissipation rate at steady state



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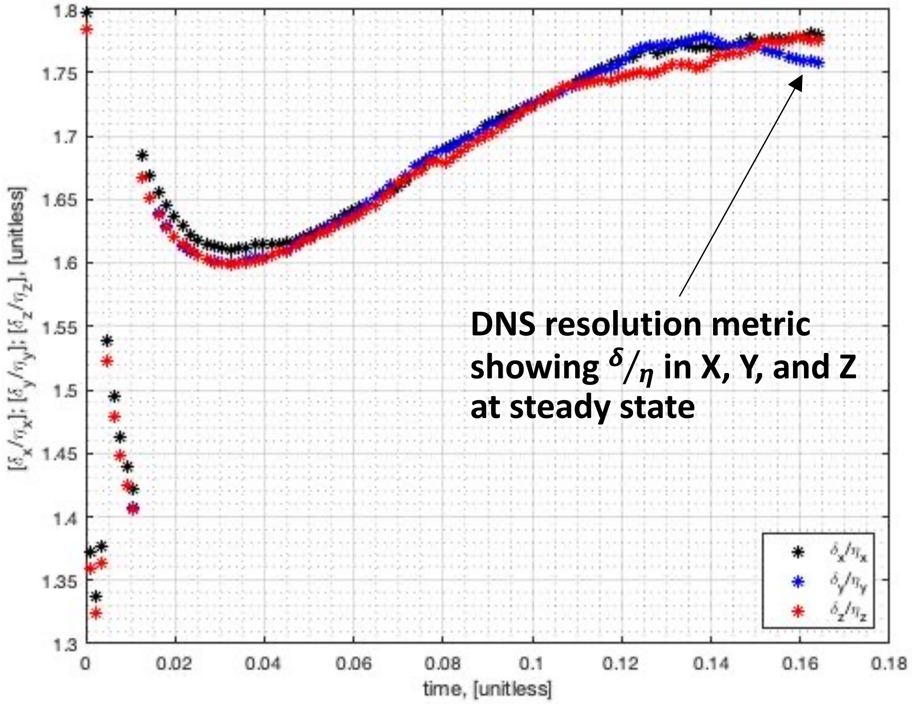
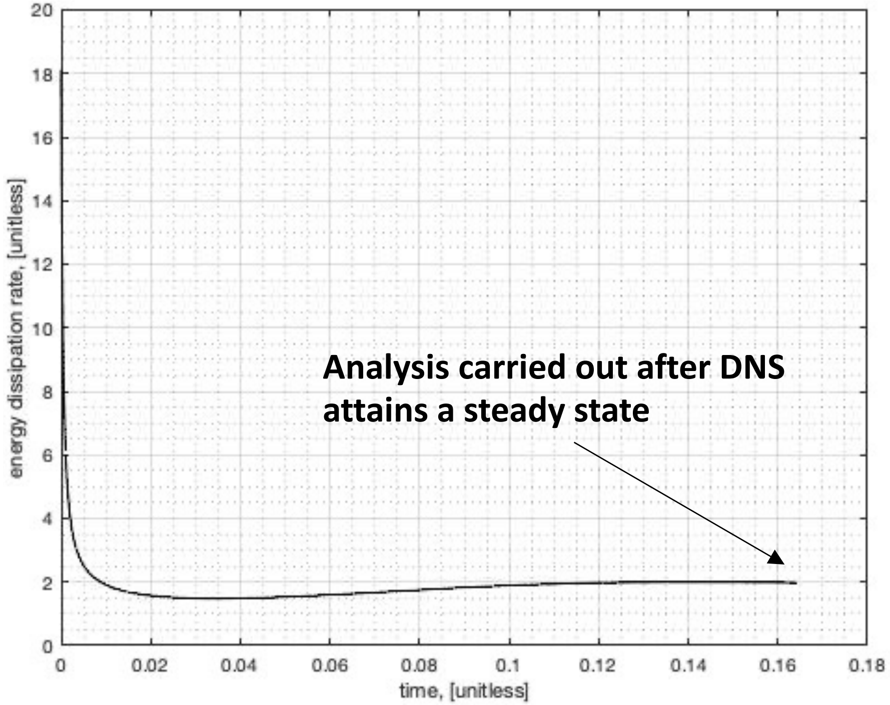
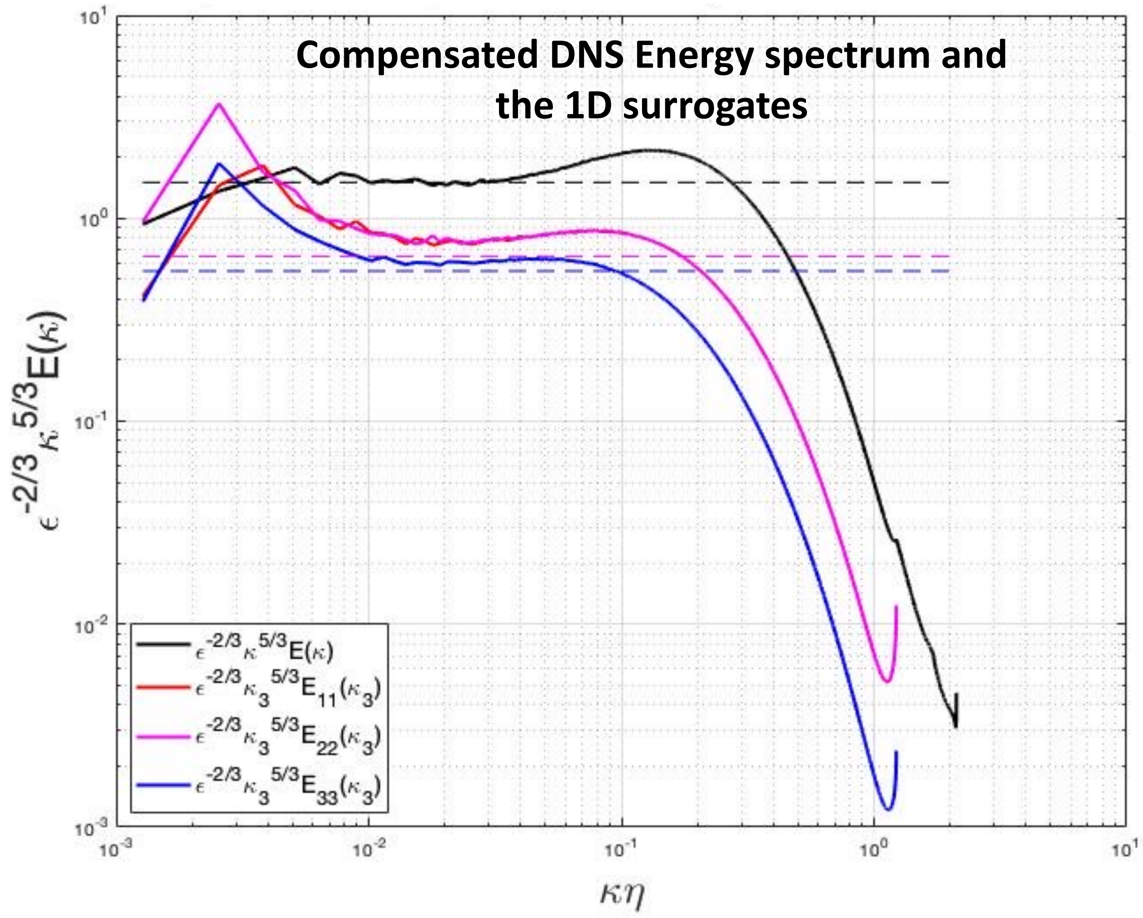


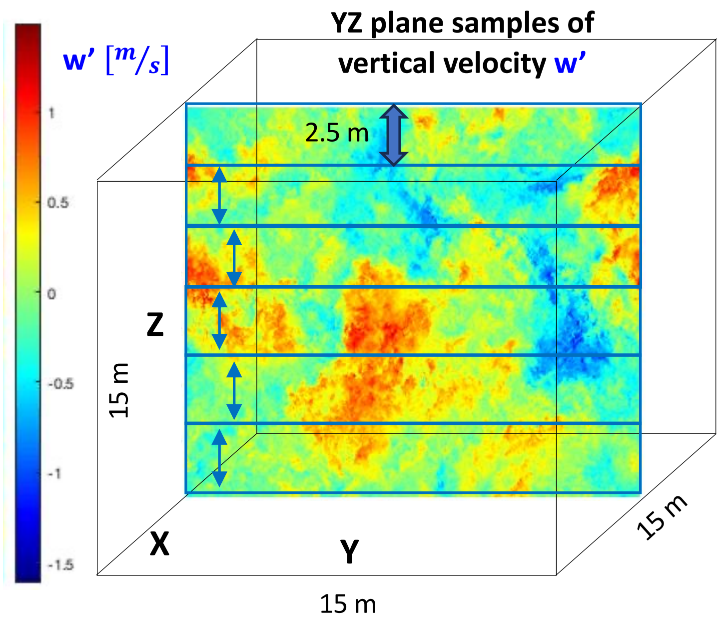


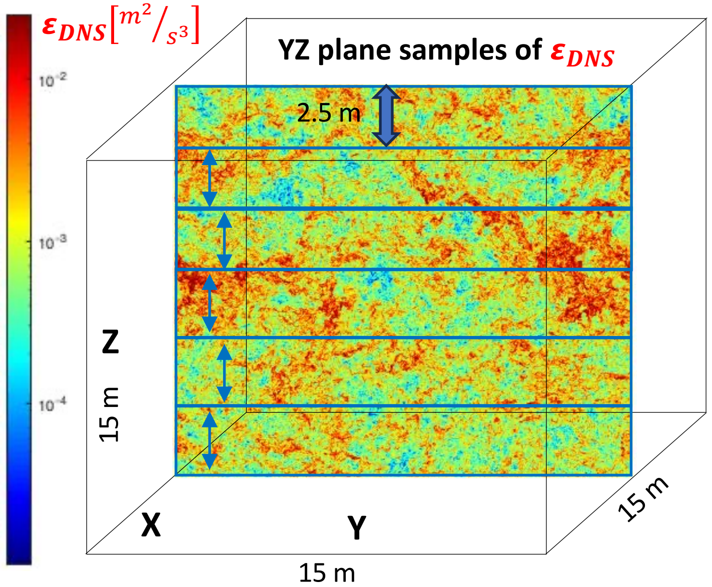
Universal Constants for 1D Energy Spectrum Surrogates

- $E(\boldsymbol{\kappa})$ - Scalar Energy spectrum; $E(\boldsymbol{\kappa}) \sim \boldsymbol{\alpha\varepsilon}^{2/3}\boldsymbol{\kappa}^{-5/3}$
- Plotting the 'compensated spectrum' $E(\boldsymbol{\kappa})\boldsymbol{\varepsilon}^{-2/3}\boldsymbol{\kappa}^{5/3}$ against normalized wavenumbers $\boldsymbol{\kappa\eta}$ results in the value for $\boldsymbol{\alpha}$ in the inertial sub-range
- Similar method is adopted to also verify the Kolmogorov universal constants for 1D longitudinal $E_{33}(\boldsymbol{\kappa}_3)$ and transverse $E_{11}(\boldsymbol{\kappa}_3)$/ $E_{22}(\boldsymbol{\kappa}_3)$ surrogate spectra; $\boldsymbol{\alpha}_L$ and $\boldsymbol{\alpha}_T$ respectively (for $\boldsymbol{\kappa}_3$ being the longitudinal direction)
- Experimentally derived values for $\boldsymbol{\alpha}$, $\boldsymbol{\alpha}_L$, and $\boldsymbol{\alpha}_T$ reported in literature are 1.5, 0.55, and 0.65 respectively
- Compensated spectra from DNS results in $\boldsymbol{\alpha}$ = 1.5, $\boldsymbol{\alpha}_L$ = 0.59, and $\boldsymbol{\alpha}_T$ = 0.7
- **DNS verification of Kolmogorov universal constants show biased estimated for $\boldsymbol{\alpha}_L$, and $\boldsymbol{\alpha}_T$**

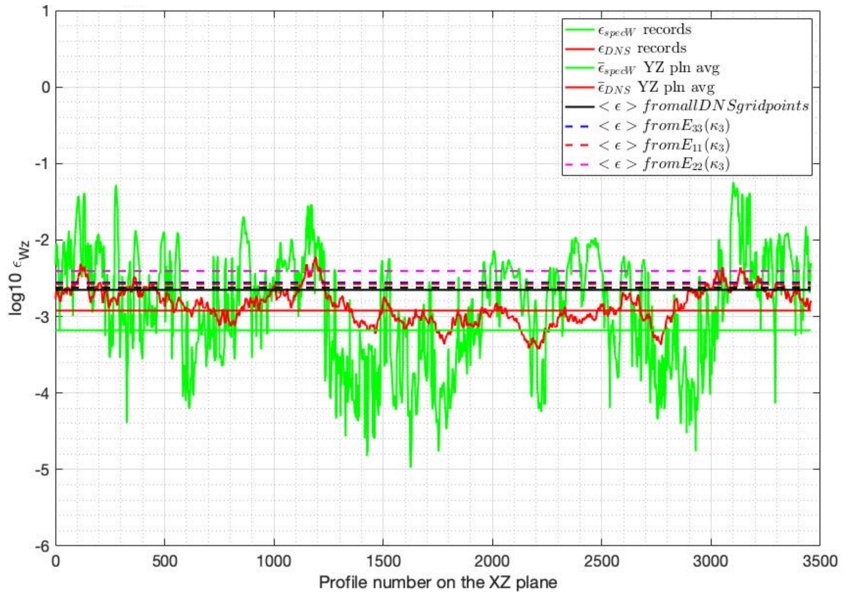


Synthetic Observations - Methodology



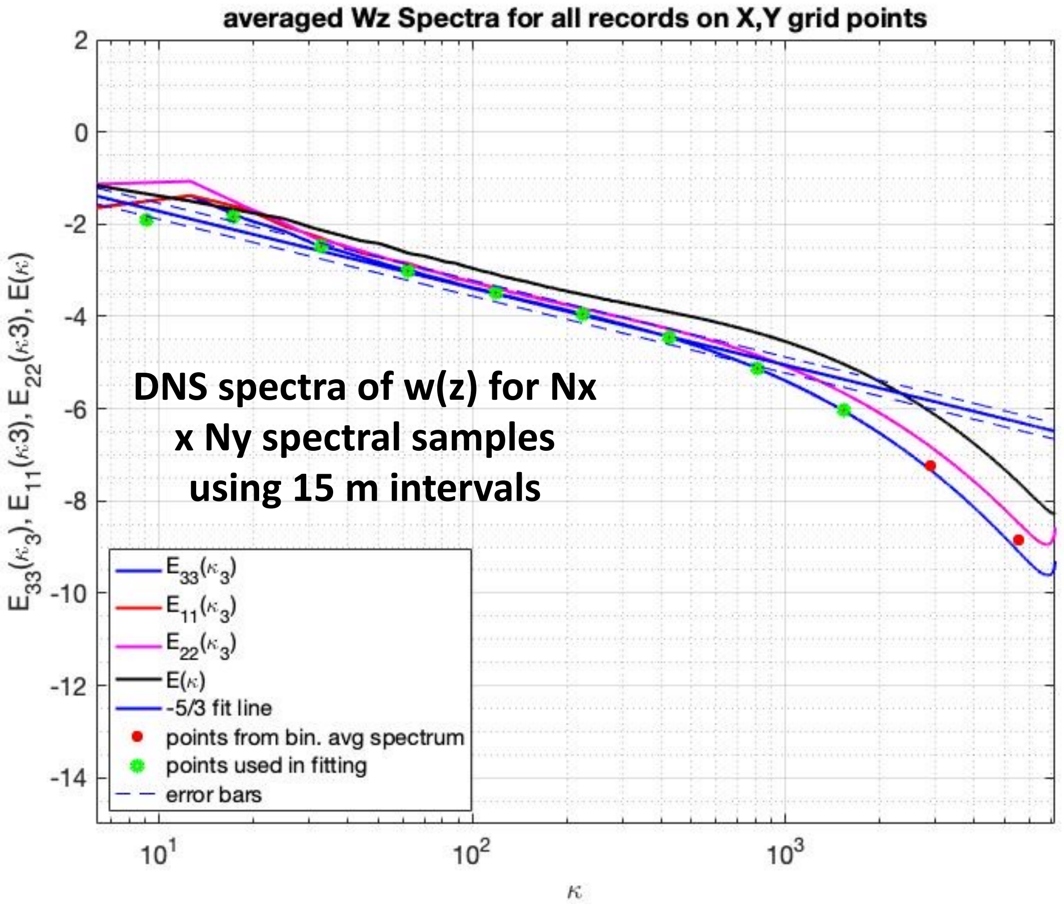


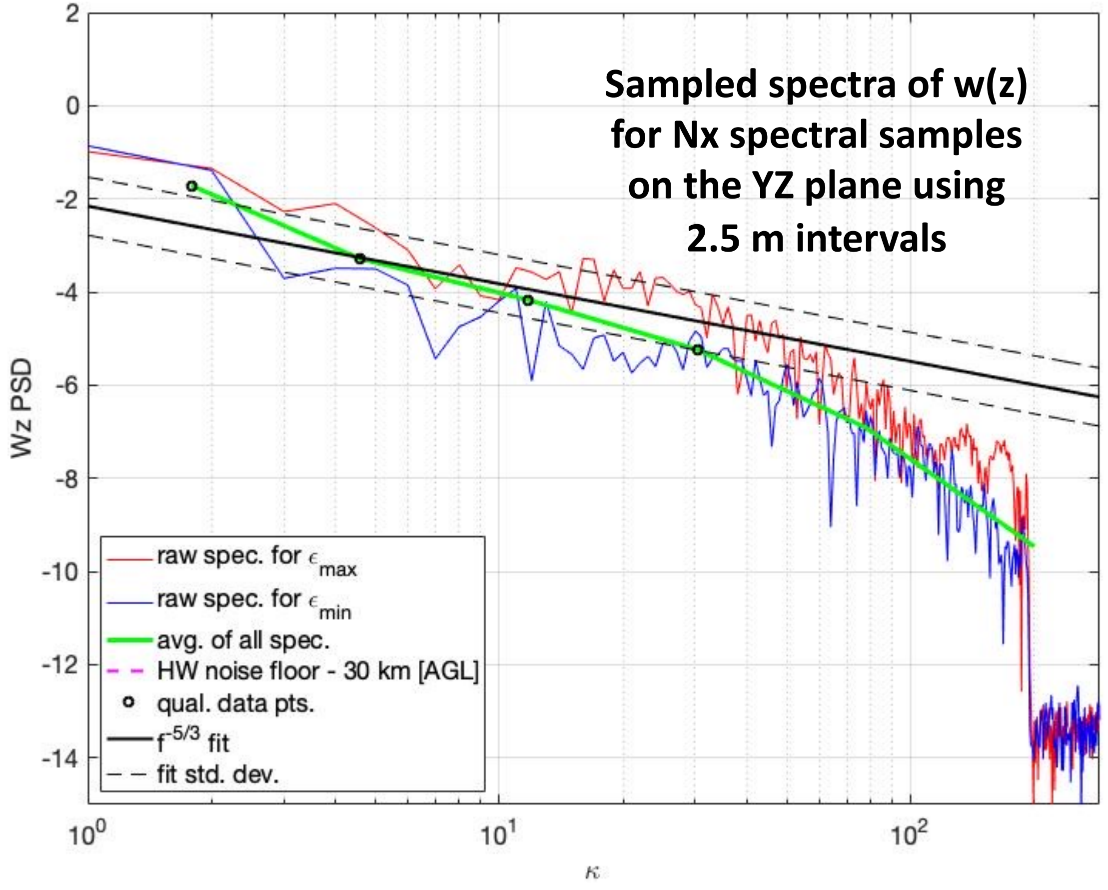
DNS non-dimensional $\hat{w}$ (on randomly sampled YZ plane) → Scaled with $\boldsymbol{\varepsilon}_M$ and $\boldsymbol{\eta}_M$ → DNS dimensional $\widetilde{w}$ (N_y x N_z samples on XZ or YZ plane) → descending data records obtained on the YZ plane → $\boldsymbol{\varepsilon}$ computed employing spectral analysis (2.5m, 6m, or 10m data records) → $\tilde{\boldsymbol{\varepsilon}}_{spec}$



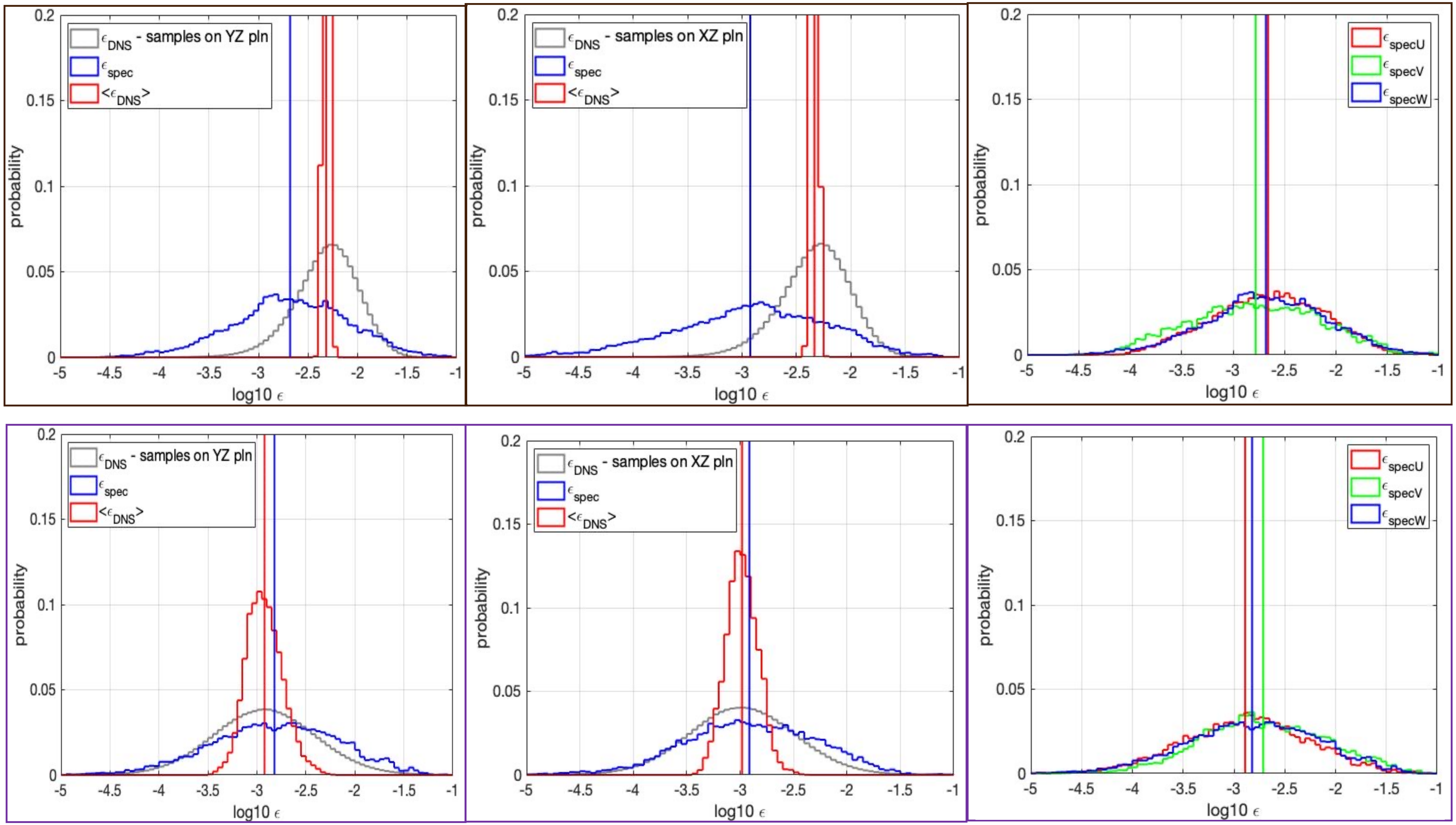
DNS non-dimensional $\hat{\boldsymbol{\varepsilon}}$ (on the same plane as $\hat{w}$) → Scaled with $\boldsymbol{\varepsilon}_M$ → DNS dimensional $\tilde{\boldsymbol{\varepsilon}}$ (N_y x N_z samples on XZ or YZ plane) → descending data records obtained on the YZ plane → $\tilde{\boldsymbol{\varepsilon}}$ averaged on data records (2.5m, 6m, or 10m data records) → $\langle \tilde{\boldsymbol{\varepsilon}}_{DNS} \rangle$

Results


averaged Wz Spectra for all records on X,Y grid points
DNS spectra of w(z) for Nx x Ny spectral samples using 15 m intervals
$E_{33}(\kappa_3)$, $E_{11}(\kappa 3)$, $E_{22}(\kappa 3)$, $E(\kappa)$
κ
$E_{33}(\kappa_3)$
$E_{11}(\kappa_3)$
$E_{22}(\kappa_3)$
$E(\kappa)$
-5/3 fit line
points from bin. avg spectrum
points used in fitting
error bars


Sampled spectra of w(z) for Nx spectral samples on the YZ plane using 2.5 m intervals
Wz PSD
κ
raw spec. for ϵ_{max}
raw spec. for ϵ_{min}
avg. of all spec.
HW noise floor - 30 km [AGL]
qual. data pts.
$f^{-5/3}$ fit
fit std. dev.

Results



Early-Stage Turbulence

- Higher mean intensity
- Narrow distr.
- Localized
- Spectral method underpredicts mean

Mature Turbulence

- Lower intensity
- Wide distr.
- Spatially more uniform
- Spectral method predicts mean

Preliminary Findings

- **Spectral measurements provide accurate distribution of TKE dissipation rates for mature (stationary homogeneous isotropic) turbulence**
  - Means of log10 $\boldsymbol{\varepsilon}_W$ and log10 $\langle\boldsymbol{\varepsilon}_{DNS}\rangle$ over 2.5m data records agree to within 0.12 of a decade
  - Variances of log10 $\boldsymbol{\varepsilon}_{DNS}$ (not averaged) are similar to variances of log10 $\boldsymbol{\varepsilon}_W$ over similar domains
  - Variance of log10 $\langle\boldsymbol{\varepsilon}_{DNS}\rangle$ (record average) is significantly smaller than log10 $\boldsymbol{\varepsilon}_W$ (spectral average over same record)---thought that this would be the closest comparison!
  - Distributions of $\boldsymbol{\varepsilon}_U$, $\boldsymbol{\varepsilon}_V$ are similar to $\boldsymbol{\varepsilon}_W$, indicating absence of significant anisotropy over the spectral data records
  - Averaging of $\boldsymbol{\varepsilon}_U$, $\boldsymbol{\varepsilon}_V$ and $\boldsymbol{\varepsilon}_W$ reduces variance in spectral estimates only marginally

- **Spectral measurements can underestimate TKE dissipation rates for unsteady (early-stage) turbulence**
  - Mean of log10 $\boldsymbol{\varepsilon}_W$ over whole planes shows a bias of 0.5 decade relative to log10 $\boldsymbol{\varepsilon}_{DNS}$, with much wider distribution
  - Distributions of $\boldsymbol{\varepsilon}_U$, $\boldsymbol{\varepsilon}_V$ are similar to $\boldsymbol{\varepsilon}_W$, indicating absence of significant anisotropy over the spectral data records
  - Bias effect is more pronounced when longer measurement intervals are used (e.g., 15 m records instead of 2 m)
  - Conjecture this is due to spatial averaging of localized strong turbulence (inherent in the spectral method), i.e., lack of homogeneity over spectral data interval length. → Suggests smaller in-situ measurement intervals are better.

Future Work

- Evaluate the influence of Taylor Frozen turbulence hypothesis.

- Compare the distribution of spectrally derived ε with second-order structure function derived ε.

- The analysis discussed so far neglects to account for the the biases introduced in the ε estimates due to observation and instrument errors such as
  - Sensitivity to balloon pendulation effects
  - Sensitivity to instrument/sensor misalignment relative to the longitudinal measurement direction (side-slip)
  - Sensitivity to instrument/sensor noise characteristics.

- Evaluate the sensitivity to wider frequency fitting ranges/time-averaging intervals.

- Investigate the applicability of Thorpe analysis to derive ε and comparisons with spectral and second-order structure function methods.

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