# Best Describing Extremes in Terms of Duration, Starting Time, and Intensity

**Eric Lu**

**(with Drs. Higgins, Mo, and Halpert)**

**CPC**
**NUIST**

# Recent Workshops on Extremes

## (Understanding, Definition, Detection)

**"Understanding and prediction of extreme events and of changes in their frequency and intensity"**

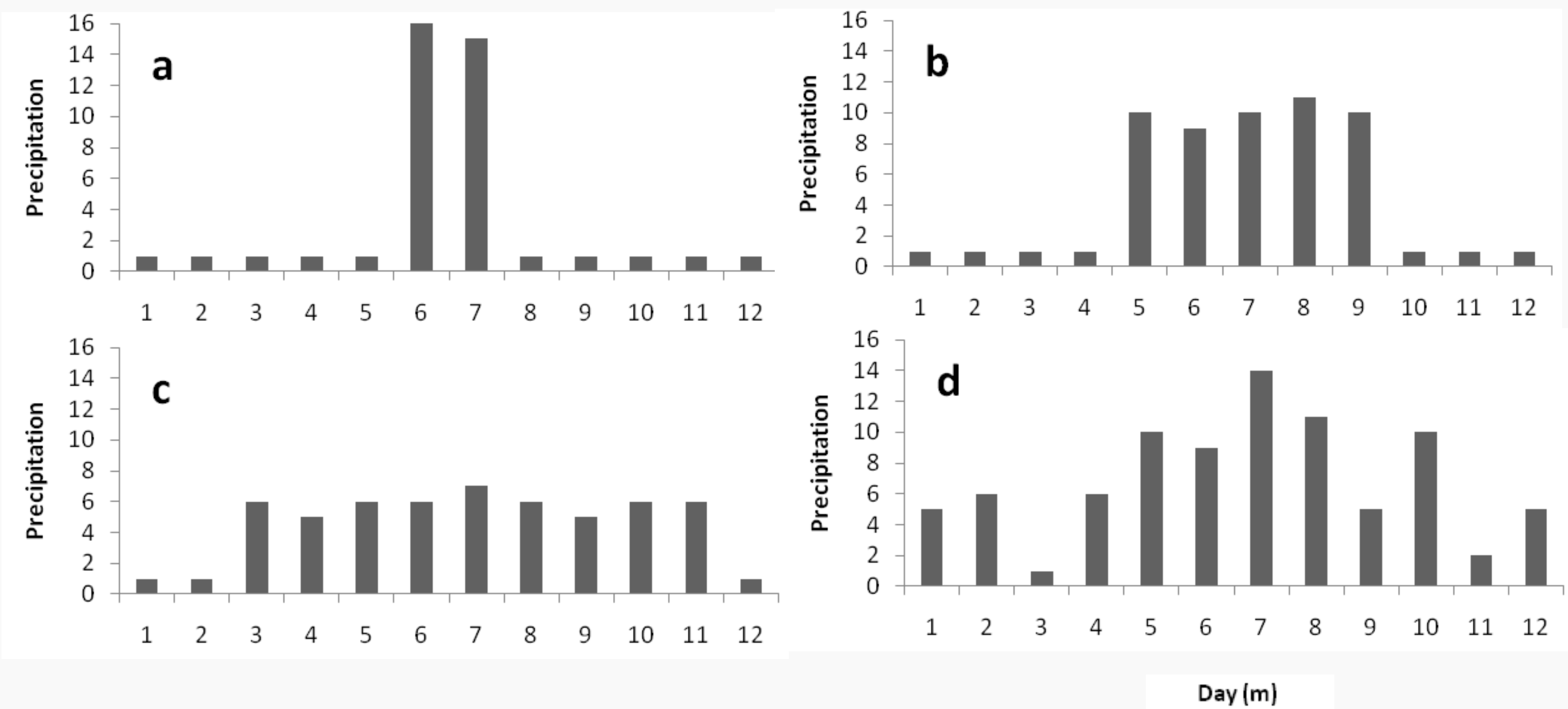
2007, Hawaii, 15th 'Aha Huliko'a Winter Workshop

**"Metrics and methodologies of estimation of extreme climate events"**

2010, France, World Climate Research Programme (WCRP) and the United Nations Educational, Scientific and Cultural Organization (UNESCO)

# Purpose

Determining the starting time and the duration of an event to make the averaged intensity of the event relatively (relative to duration) the strongest, compared with other events with other starting times and durations, so the event best becomes an extreme.



## How does "Extreme" Intensity vary with Duration?

### Principle

$$\frac{dI_e}{dT} < 0$$

$$\frac{d(I_e T)}{dT} > 0$$

## Discount rates: unit price, pieces, total payment

$$\frac{dI_e}{dT} < 0$$

**buyer**

$$\frac{d(I_e T)}{dT} > 0$$

**seller**

# Equation

$$a = -\frac{1}{I_e}\frac{dI_e}{d(\ln T)}$$

$$0 < a < 1$$

## “Extreme” Intensity ~ Duration

$$T = n\Delta T$$

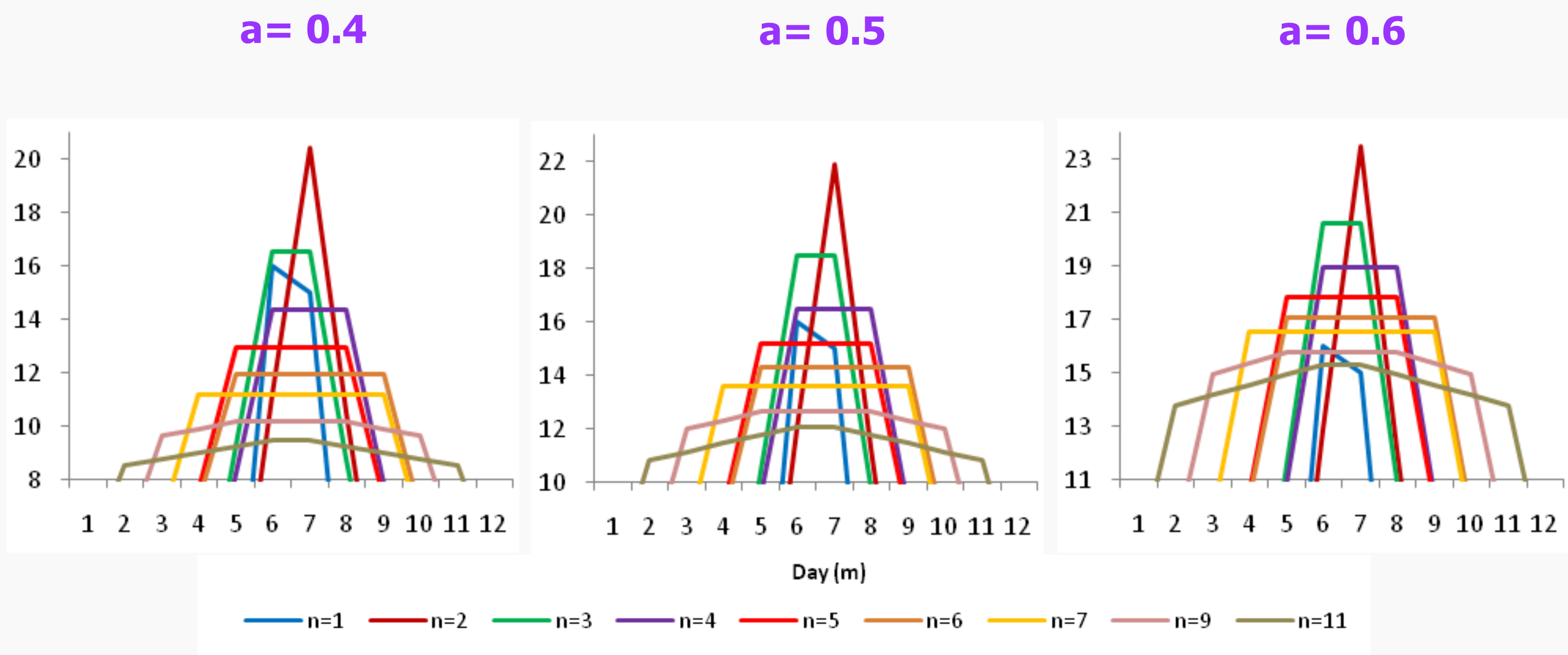
**integration**

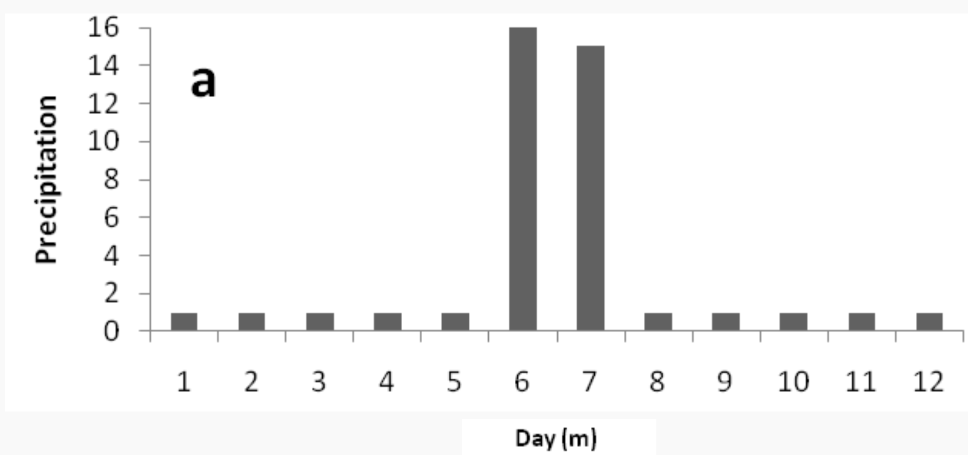
$$I_e(n) = I_e(1)\, n^{-a}$$

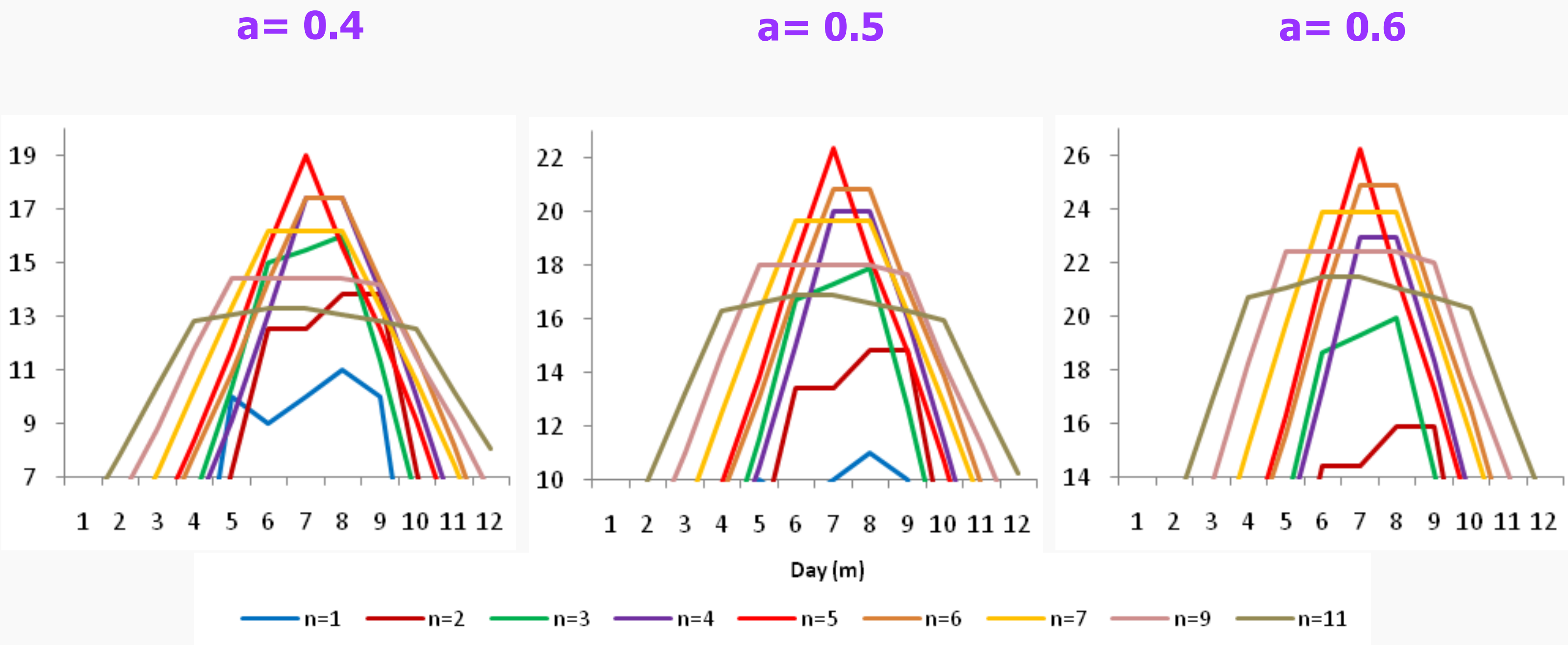
## Comparison among Durations

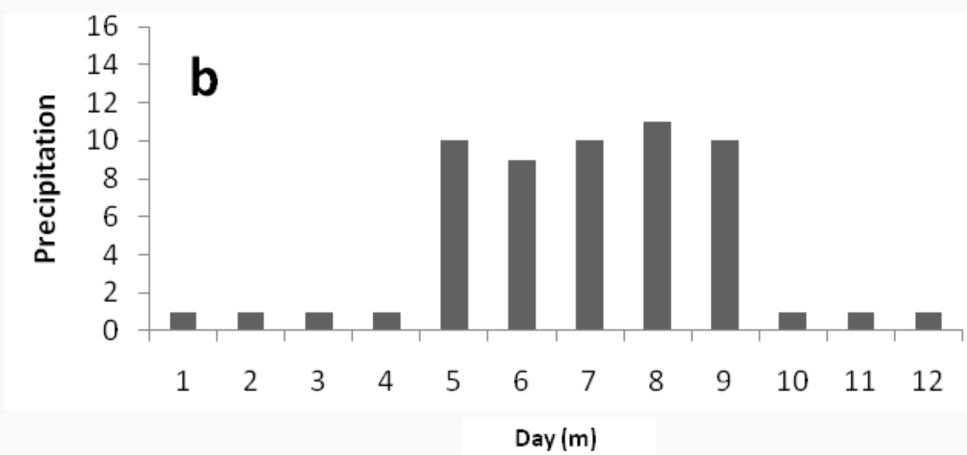
### Relative Intensity

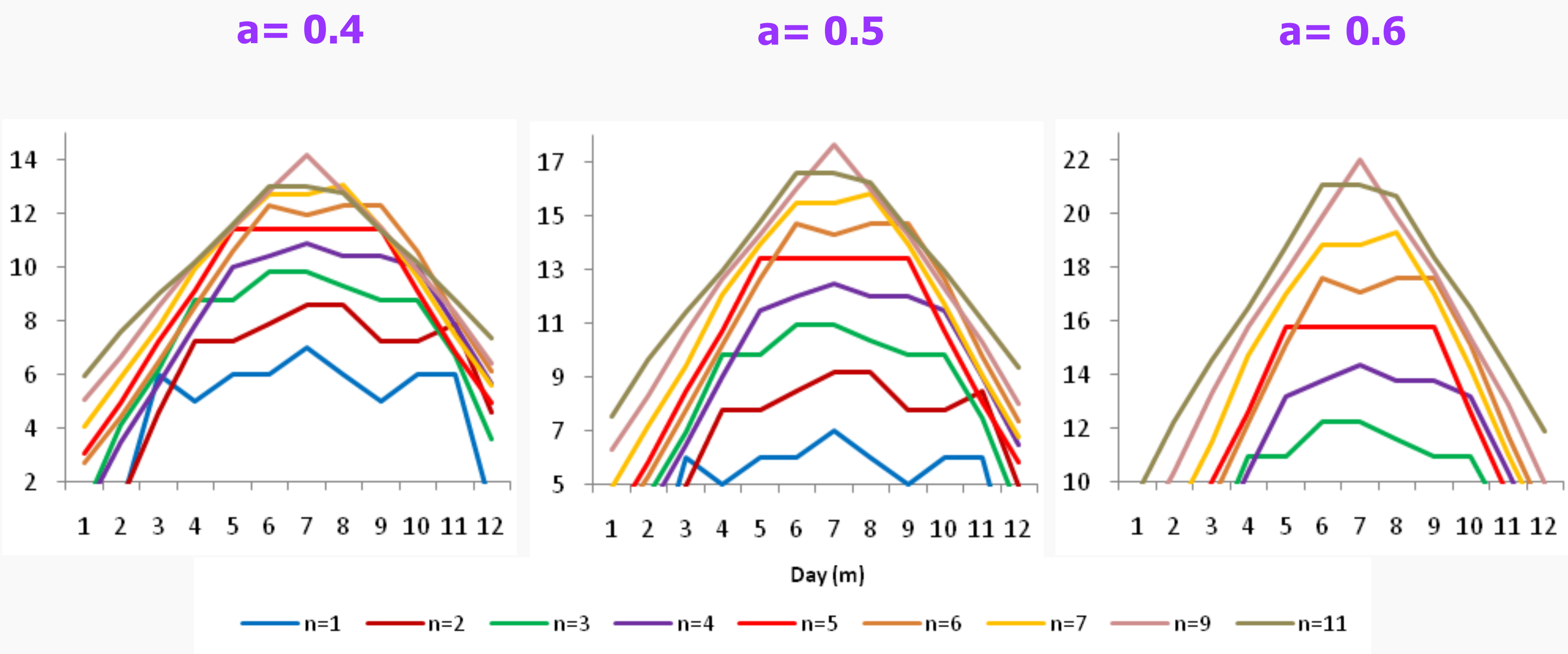
$$R(n,m) \equiv \frac{I(n,m)}{n^{-a}} = n^{a} I(n,m)$$

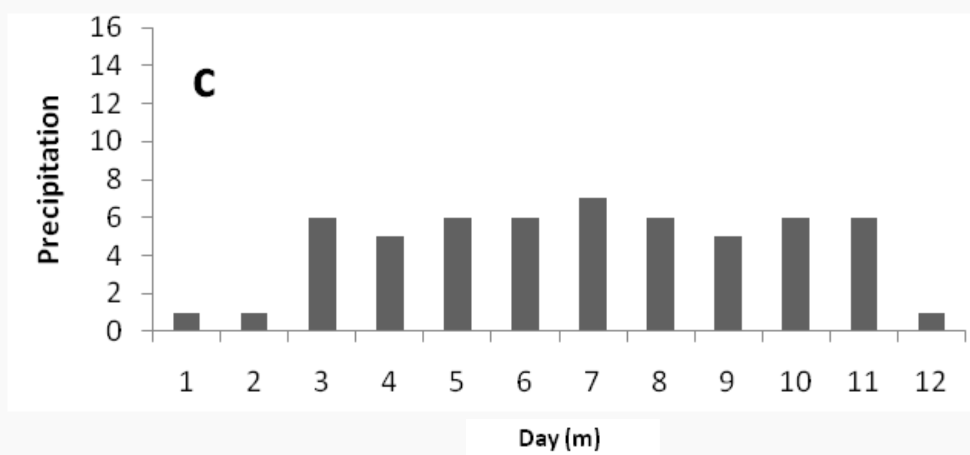

a= 0.4
a= 0.5
a= 0.6
Day (m)
n=1
n=2
n=3
n=4
n=5
n=6
n=7
n=9
n=11


a
Precipitation
Day (m)


a= 0.4
a= 0.5
a= 0.6
Day (m)
n=1
n=2
n=3
n=4
n=5
n=6
n=7
n=9
n=11


b
Precipitation
Day (m)

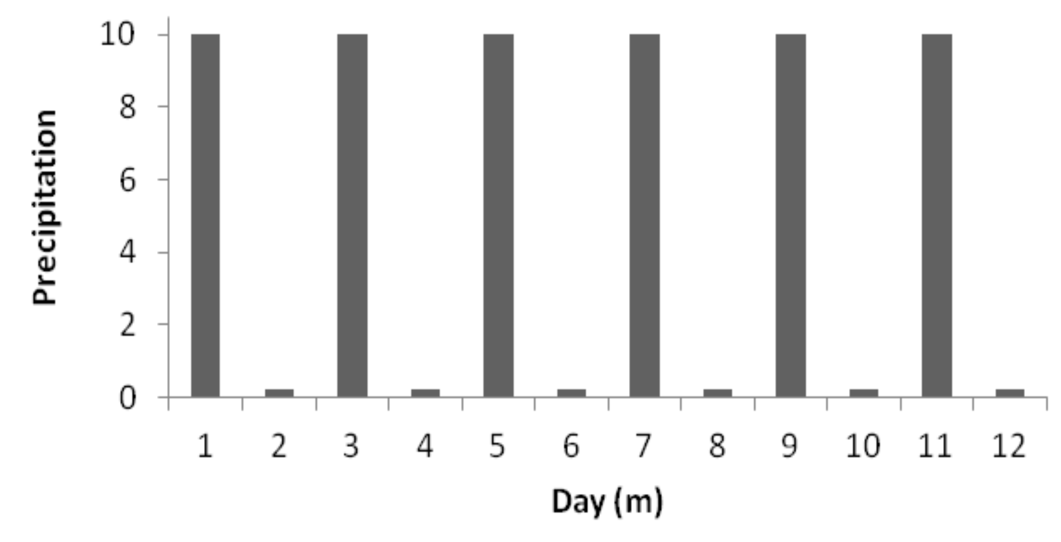

a= 0.4
a= 0.5
a= 0.6
Day (m)
n=1
n=2
n=3
n=4
n=5
n=6
n=7
n=9
n=11


c
Precipitation
Day (m)

# Multi-year Data

$n$ $I_e(n)$ "top 5%"

**Spectrum of intensity over duration may have irregular structure**



## Parameter 'a' – regressed from data

$$I_e(n) = I_e(1)\,n^{-a}$$

**Regression**

$$\ln I_e(n) = -a\ln n + c$$

# Summary

- If no 1-day extreme, look at other durations (multi-day extremes )
- ‘Extreme’ intensity – duration relation is the key
- For other quantities (T) and timescales (hourly)
- “Top 5%” – spectrum of intensity may have irregular structure
- Follow-on study - use regression to calculate parameter ‘a’

# Monitoring Drought at Daily Scale

## Change rate of Flood Extent

$$\frac{df(t)}{dt} = -bf(t) + P(t)$$

## Integration

$$\left[e^{bt} f(t)\right]_{-\infty}^{0} = \int_{-\infty}^{0} e^{bt} P(t)\,dt$$

$$f(0) = \int_{-\infty}^{0} e^{bt} P(t)\,dt$$

## Using daily data

$$f_0 = \Delta t \sum_{n=0}^{\infty} a^n P_n$$

## Truncation

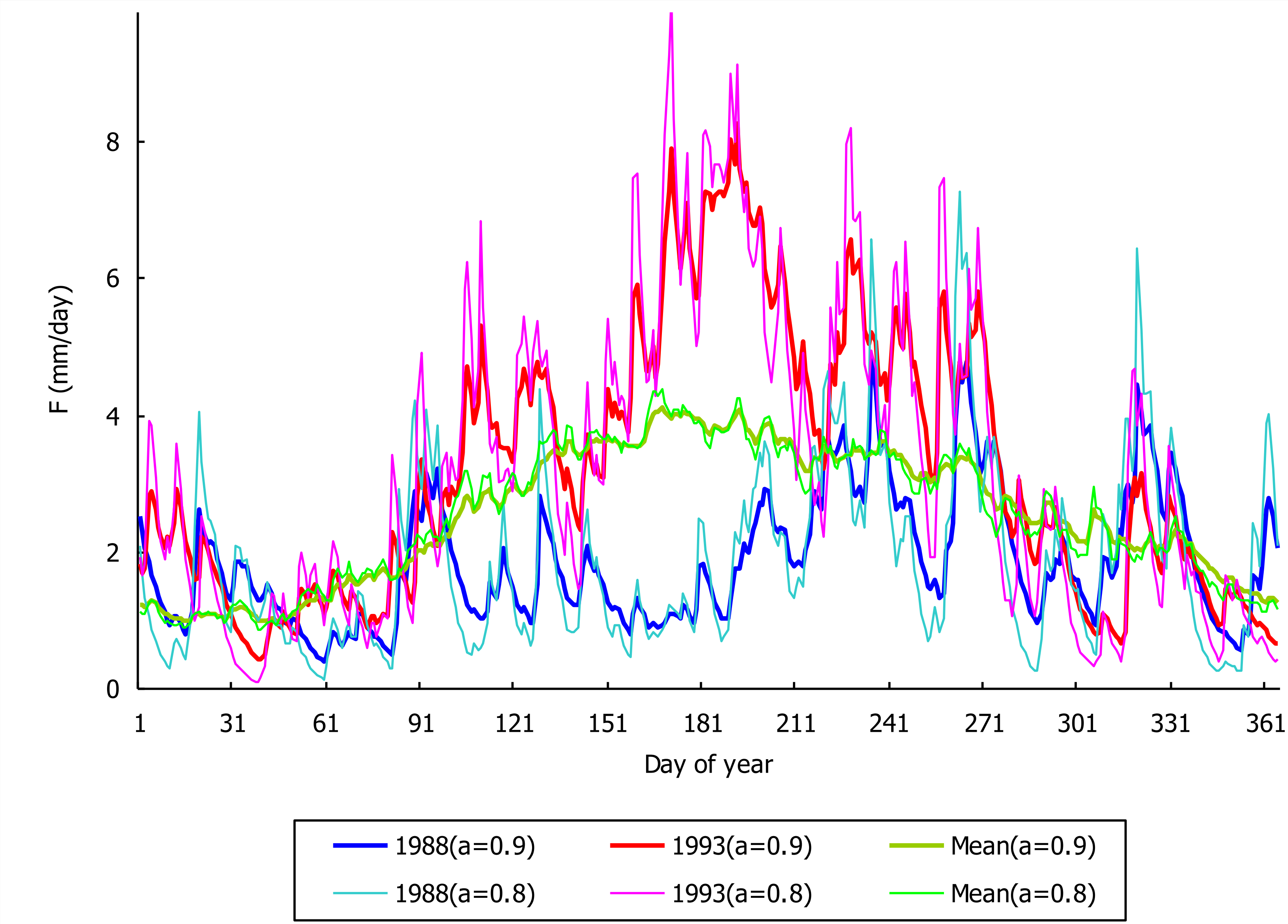
$$f_0 = \Delta t \sum_{n=0}^{N} a^n P_n$$

## Weighted Average of Precipitation

$$WAP = \sum_{n=0}^{N} a^n P_n \Big/ \sum_{n=0}^{N} a^n$$

$$= \boxed{(1-a)\sum_{n=0}^{N} a^n P_n}$$

# 1988, 1993, and 1980-2004 Mean



# Thanks!