Analysis of a Simple Liouville Theory Based Approach to SEP Hazard Specification

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Introduction

We analyzed a simple Liouville theory based approach to solar energetic particle (SEP) hazard specification which assumes a quasi-static magnetic field. We used the method with six rigidity cutoff models, including a control model that set the cutoff rigidity to zero. We compared the model results to observations from the Highly Elliptical Orbit (HEO) satellite, Combined Release and Radiation Effects Satellite (CRRES) and the Van Allen Probes missions. The events were identified automatically from GOES data during each satellite's mission as any period where the >10 MeV integral flux was greater than 10 pfu.

Liouville Mapping Approach

Liouville's theorem [1] is a powerful statistical theorem that states that the phase space density along a dynamical path remains constant. This result is often used to simplify difficult calculations

When we restrict ourselves to the case where,

- electric fields are negligible, and
- the interplanetary flux is isotropic,

we get a further simplification. In this case the energy spectrum at locations inside the magnetosphere will be the same as the interplanetary spectrum for particles with energies, *E*, that are above the cutoff energy, $E_C(\eta)$, and it will be zero for particles with energies below $E_C(\eta)$ [2]. Note that $E_C(\eta)$ depends on a particle's direction of approach, *η*.

Using GOES observations we calculate the integral flux, F_{local}, at the target location as,

$$F_{local} = 4\int_{\eta=0}^{\pi}\int_{E=(\max(\{E_T,E_C(\eta)\})}^{\infty} f_{IP}(E)\,\mathrm{d}E\,\mathrm{d}\eta$$

E_T = threshold energy, above which the fluxes become hazardous, f_{IP} = interplanetary differential flux. We interpolated GOES corrected integral fluxes to the maximum of E_T or E_C for each direction, η, and integrated over direction. The GOES instruments used depended on the date. They were either the Energetic Particle Sensor (EPS) or the Energetic Electron, Proton and Alpha Detector (EPEAD) [5].

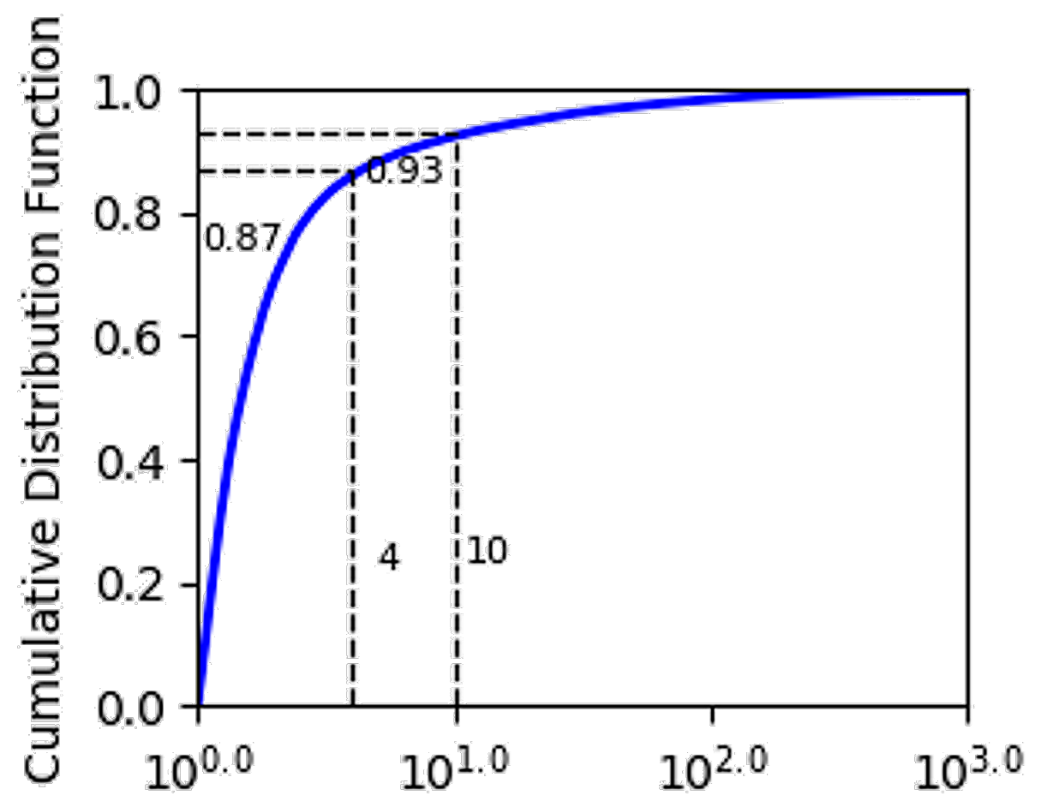
Statistical Measures

We wanted to know how often these models were within a factor of 4 or 10 of the observations. So we based our investigation on the **matching ratio**:

$$R \equiv \begin{cases} Q, & \text{if } Q \geq 1 \\ 1/Q & \text{otherwise} \end{cases}$$

Where the **accuracy ratio**, *Q*, is defined as:

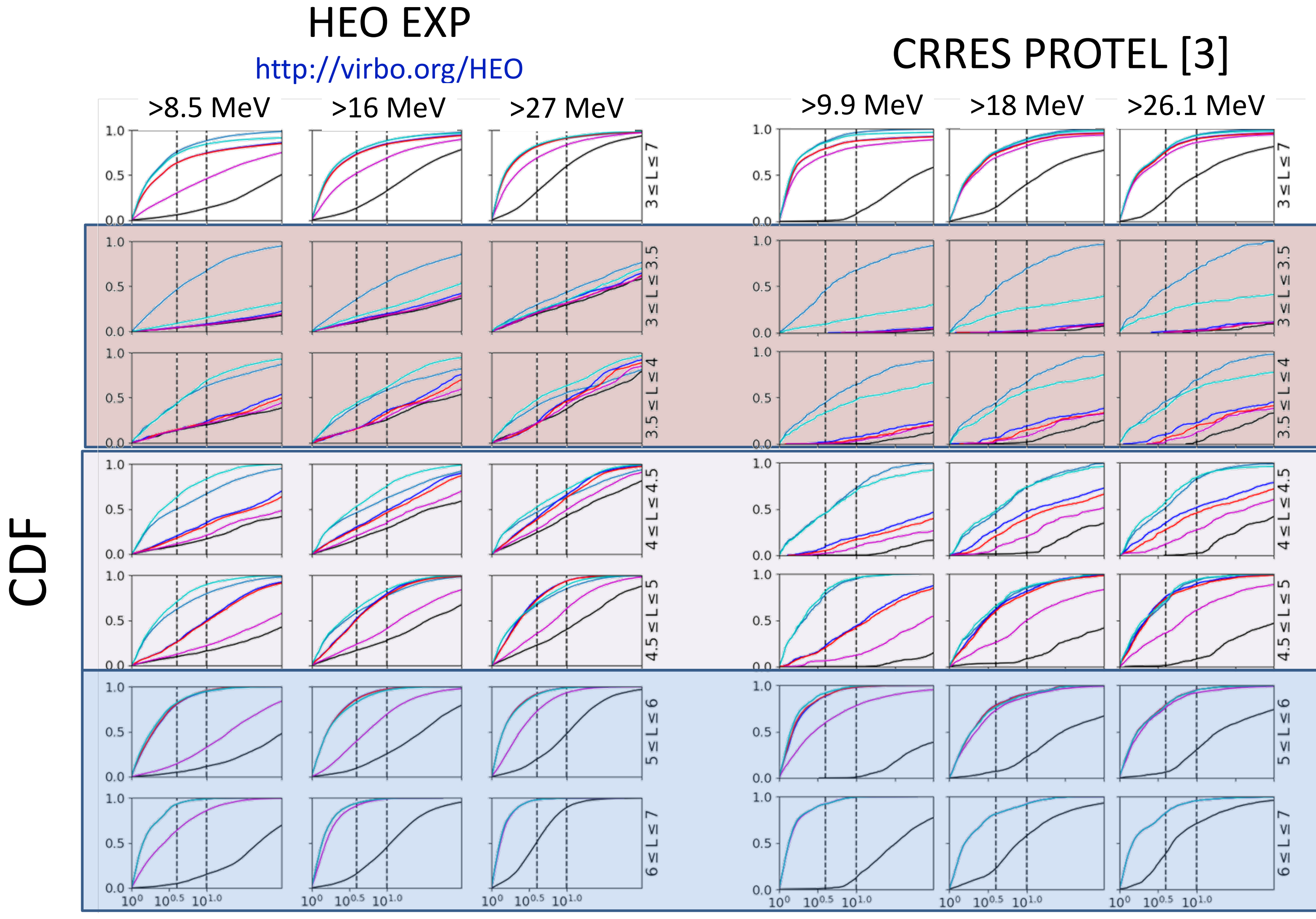
$$Q \equiv \frac{F_{local}^{mod}}{F_{local}^{obs}}$$



The cumulative distribution function of *R*, CDF(*R*), is the fraction of matching ratios that are within a factor of *R*

Ideally, the CDF(R) rises sharply until it reaches a value of 1. The example above is reasonably good, given its logarithmic x-axis. 87% of the model results are within a factor of 4 from the observations and 93% within a factor of 10.

Results



L ≥ 3

3 ≤ L < 4

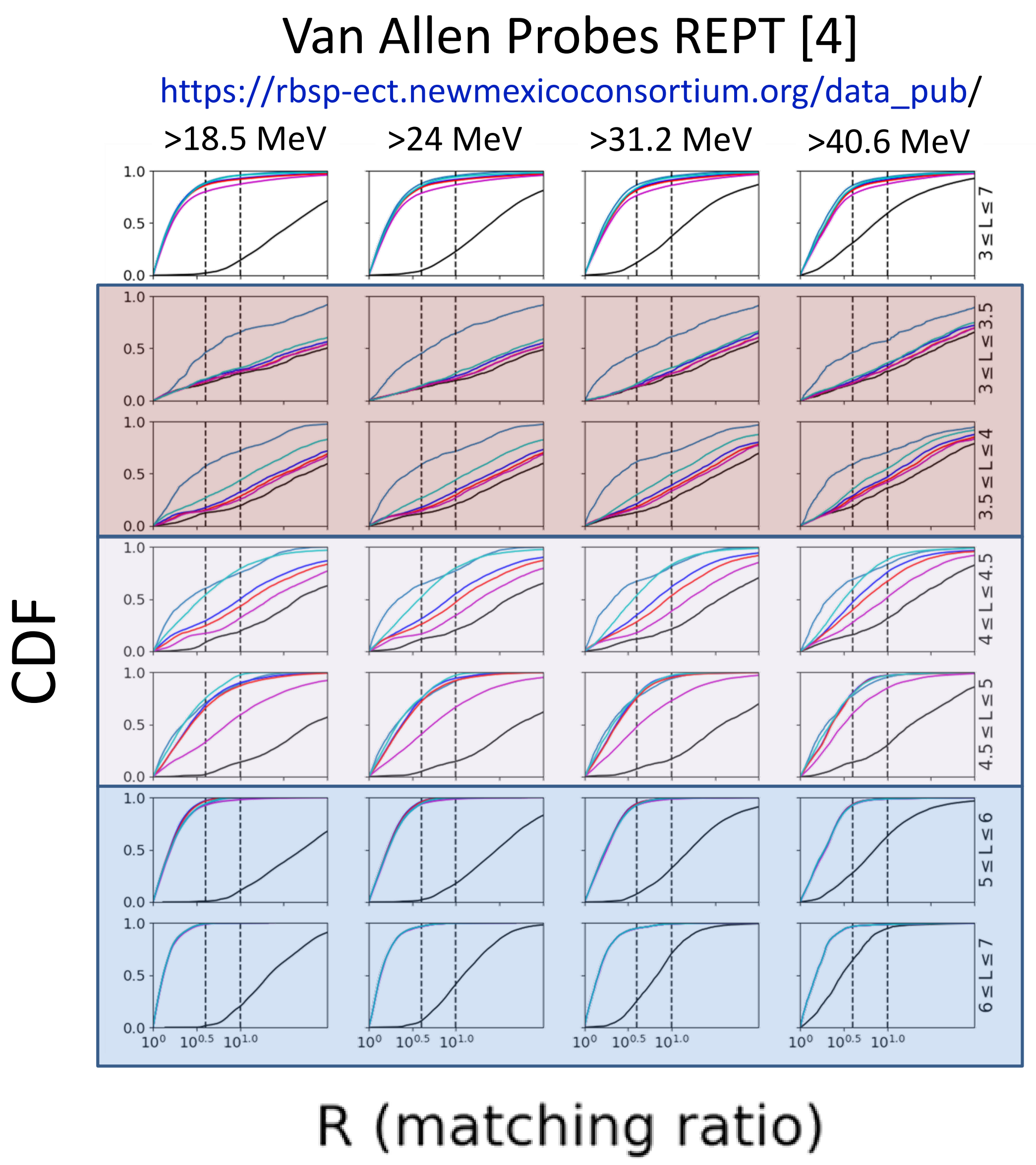
Control better than models. No good results.

4 ≤ L < 5

Significant spread in model performance. Control or SNO usually top performer.

L ≥ 5

Most models performed similarly. Dipole and Smart and Shea perform poorly.



The two dotted lines in each plot mark R=4 (left) and R=10 (right).

Rigidity cutoff Models

Ogliore 1 & Ogliore 2 models: Empirical fits to SAMPEX data. [6]

Smart and Shea: Based on table of pre-computed reverse trajectory traces in Tsyganenko 89+Boberg ext. [7]

Selesnick-Neal-Ogliore (SNO): Ogliore 2 model, but with Kp dependence from Neal et al. [8,9]

Dipole: Dipole cutoff model.

Control: No cutoff model – flux at GOES assumed to be local flux.

Note: Except for the control and Smart and Shea models, all models have been modified. They use L calculated using the Olson-Pfitzer Quiet Time model [10] to extend their results to high altitudes.

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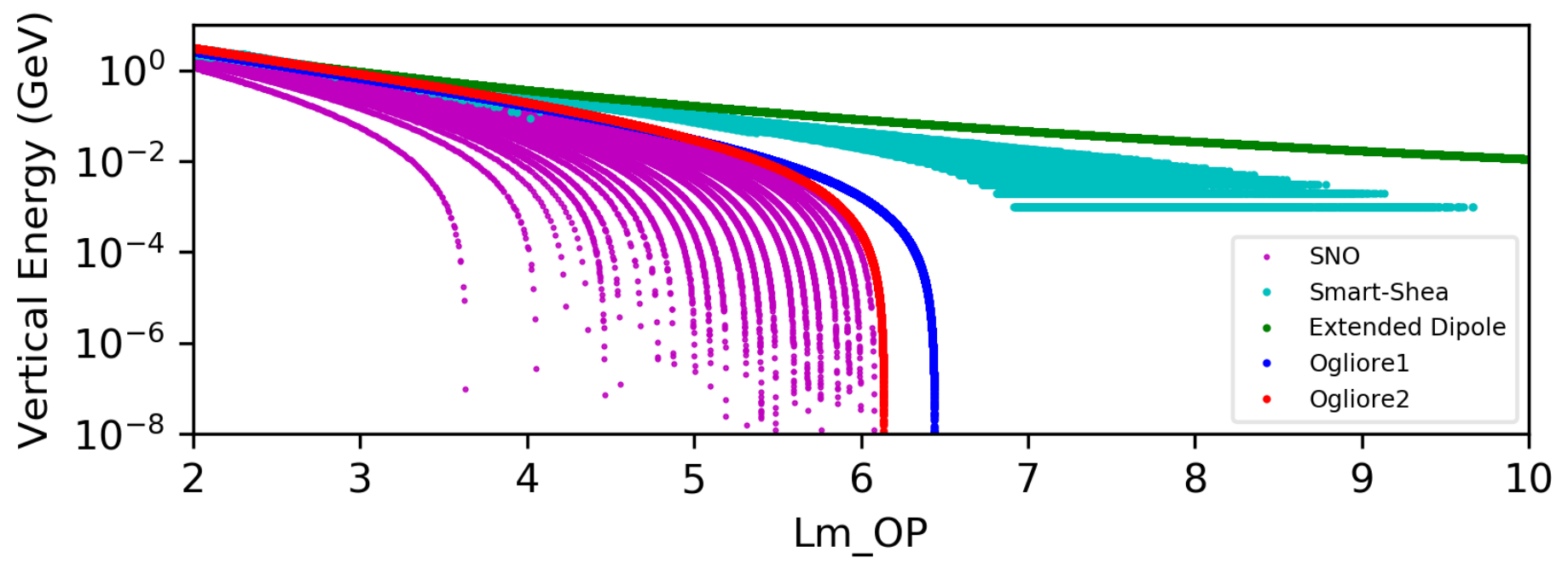
Discussion and Conclusions

- The control model performed the best overall.
- For L>5 most models performed similarly and may not be necessary.
- In the region 4 ≤ L ≤ 5, there was significant model differentiation. Only the SNO model provided any benefit over the control model; that was seen along the HEO orbit.
- The SNO model's overall success seems to have been the result of its strong Kp dependence and/or low cutoff energies.
- The SNO model's success also suggests that a better static model may be a benefit between 4 ≤ L ≤ 5.
- Neither the control, static nor quasi-static models specified fluxes below L=4 very well. Magnetospheric dynamics are required to model the physics that drive particles into this region.



Vertical cutoff energy, color-coded by model, as a function of L and calculated along the HEO orbit during SEP events. The large variations in the SNO and Smart and Shea models are due to their dependence on Kp.

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