

Reconstruction of 3D Electron Density of the Global Corona by Tomography from Multi-vantage Observations

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Introduction

- Objectives of the 3D Coronal Density Reconstruction
 - Understand the 3D structure and magnetic fields of solar corona
 - Understand the short-term and long-term variability of the inner/middle corona
 - Understand the solar wind's origin
 - Provide crucial constraints on the inner boundary for solar wind model to improve space weather prediction
 - Provide coronal background model to understand the origin of shocks and radio bursts produced by flare/CME

- Task of our Work
 - Develop and apply the tomography technique to reconstruct the 3D electron density from multiple vantage-point pB observations by STEREO/COR1 and LASCO/C2 (current)
 - Extended to the outer corona using STEREO/COR2, LASCO/C3, and Solar Orbiter/METIS
 - Expanded to future missions such as PUNCH/NFI and CODEX

Regularized Tomography

- Linear inversion problem

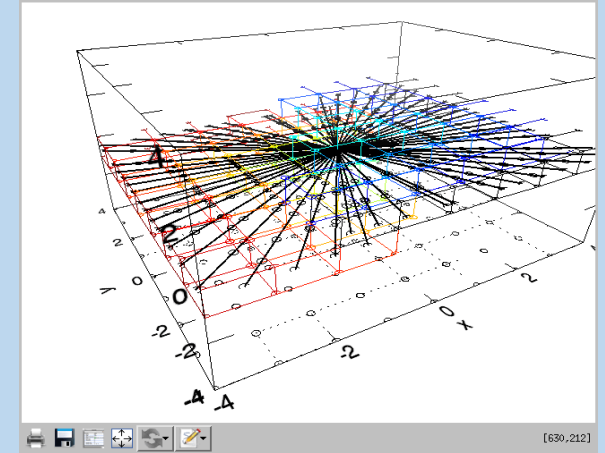
$$pB(\rho) = \int_{\text{LOS}} K(\rho, r) N(s) ds \quad \longrightarrow \quad \mathbf{AX} = \mathbf{Y}$$

Where $\mathbf{X}=[n_1, n_2, \dots]^T$ unknown density, $\mathbf{Y}=[b_1, b_2, \dots]^T$ data, and Matrix \mathbf{A} contains the known coefficients depending on the geometries and Thomson scattering

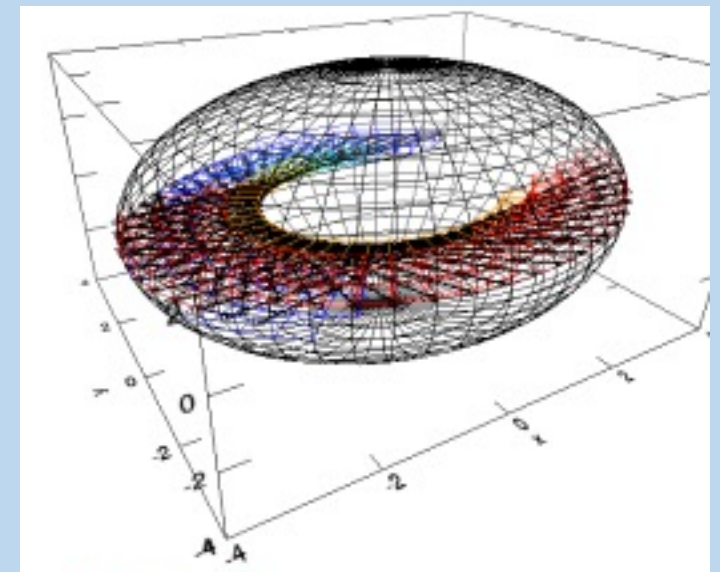
- Solving Linear problem by minimization of loss function with regularization (Frazin 2000; Frazin & Janzen 2002; Kramar +2009)

$$F = \|\mathbf{AX} - \mathbf{Y}\|^2 + \mu \|\mathbf{RX}\|^2$$

- $\|\mathbf{RX}\|^2$ represents the smoothing of the solution, e.g., L2 – norm of 1st order or 2nd order derivatives
- Regularization term used to reduce the noise effect and make the solution stable and unique



Cartesian grid: $N=9 \times 9 \times 9 = 729$ $M=28$ rays



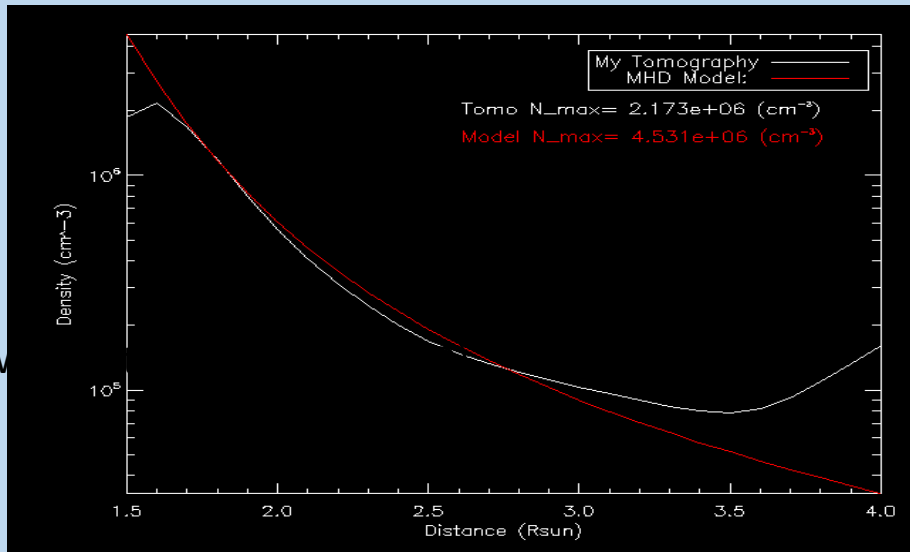
Spheric grid: $N_\phi=51$, $N_\theta=26$, $N_r=6$, $M=28$ rays

**Improvement of regularization method:
Adding a weighting factor**

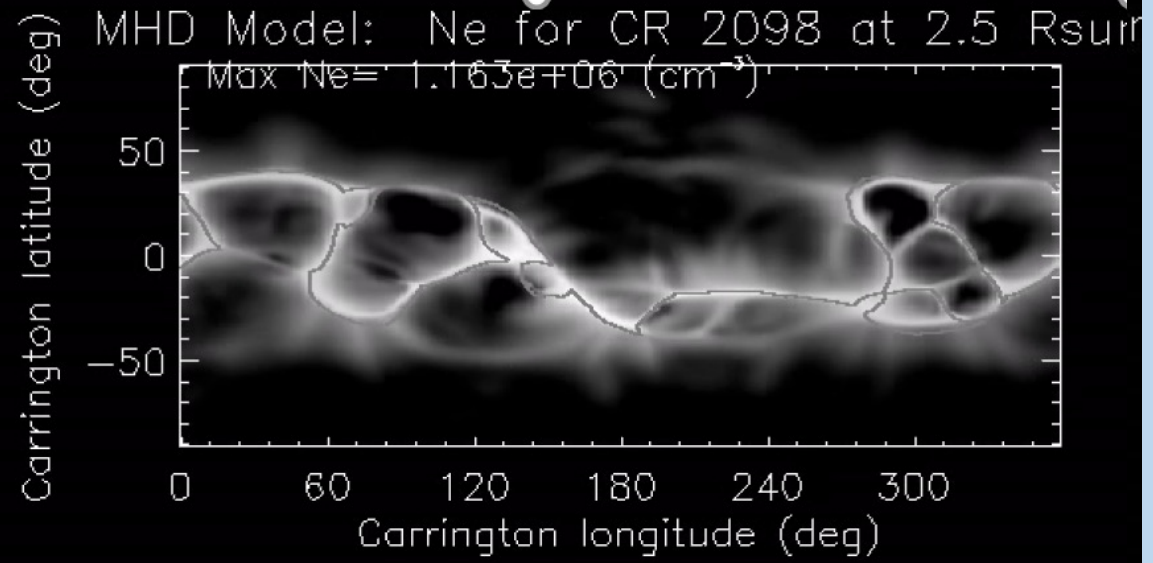
- to avoid overly-smoothing of the high-density structure at lower heights
- Increase solution stability for high heights

$$F = \|AX - Y\|^2 + \mu \|R_W X\|^2$$

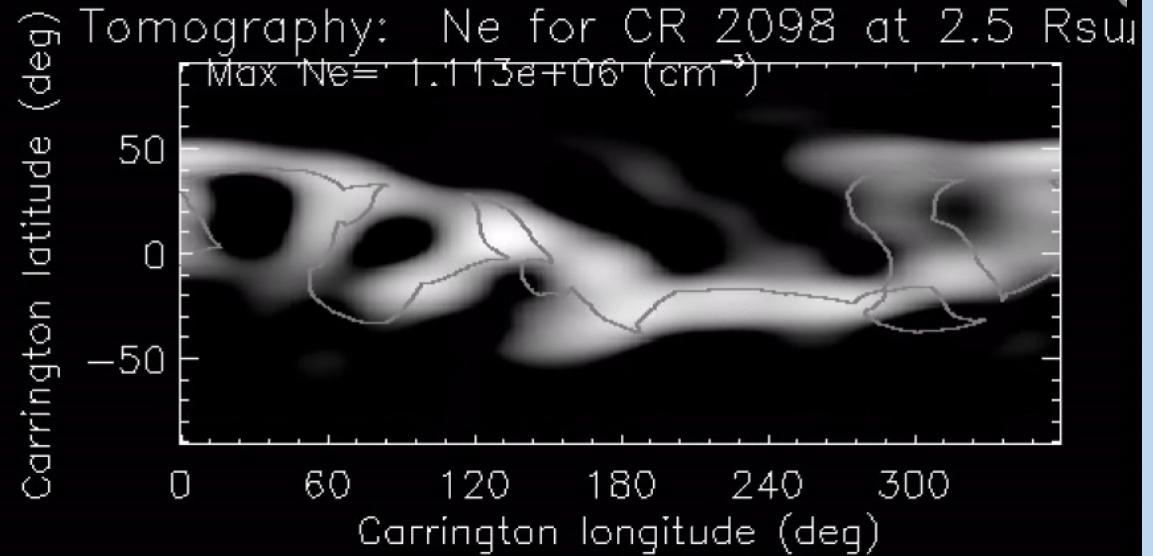
Where $R_W = WR$, and W is a diagonal matrix with $W_k = 1/N_{bg}(r_k)$, where $N_{bg}(r)$ is global average of SSPA model



Comparison with thermodynamic MHD solution by PSI (Lionello+2009)



MHD model by PSI



Regularization with a weighting factor

- Minimization with 0th-order regularization

- 0th-order regularization (ridge regression)

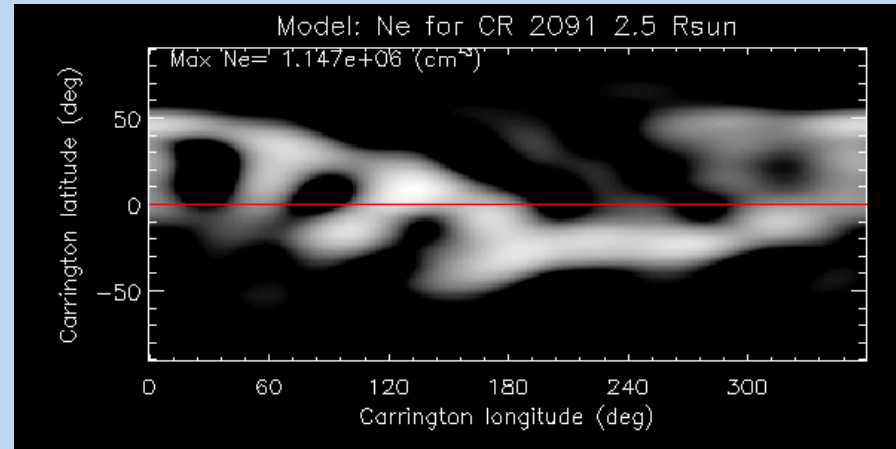
$$F = \|AX - Y\|^2 + \mu \|X\|^2$$

From $\frac{\partial F}{\partial X} = 0$, obtain

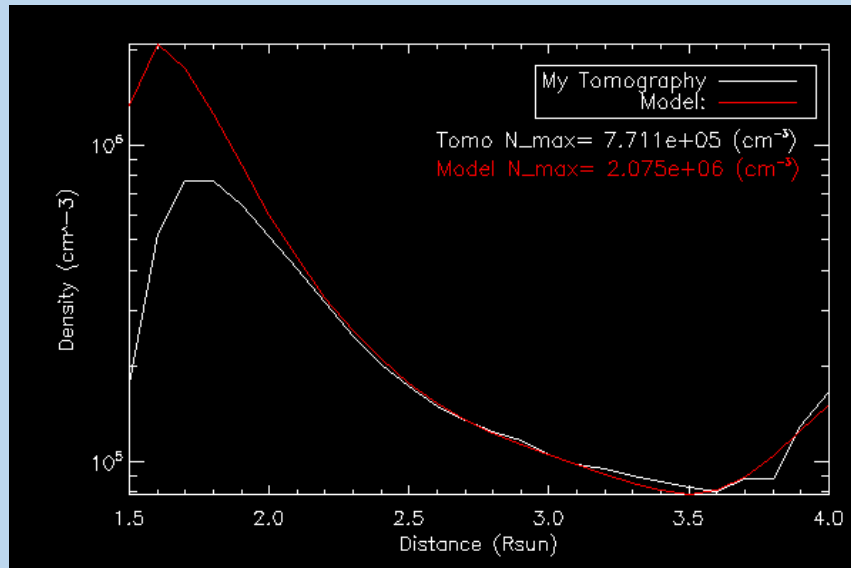
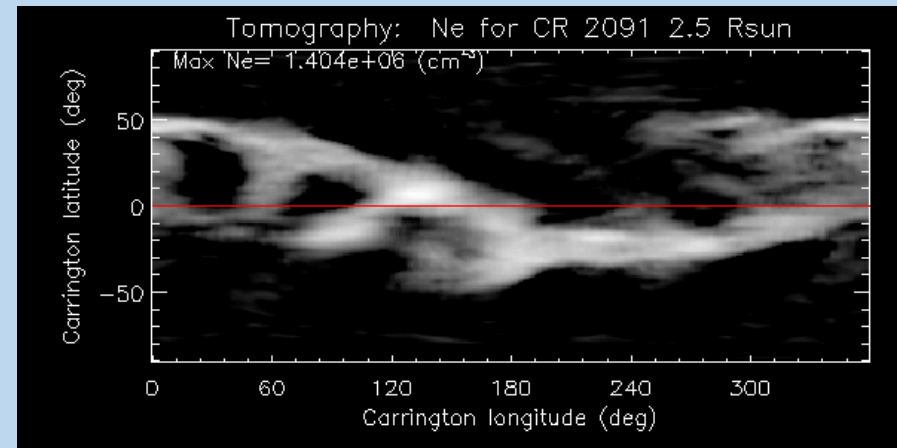
$$(A^T A + \mu I) X_\mu = A^T Y \quad \text{where } R^T R = I, \text{ is a unit matrix}$$

$$MX_\mu = b$$

$N=128^3$, 2nd-order smooth, weight= $1/N_{bg}(r)$



$N=64^3$, 0th-order smooth, no weight



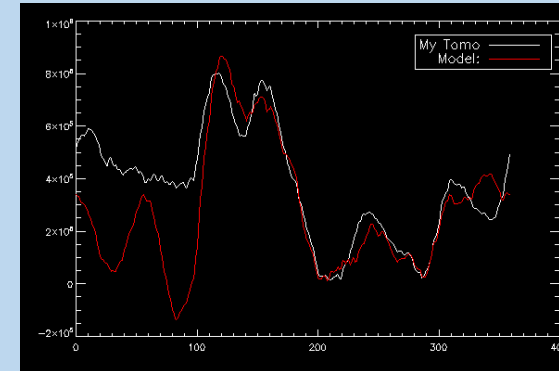
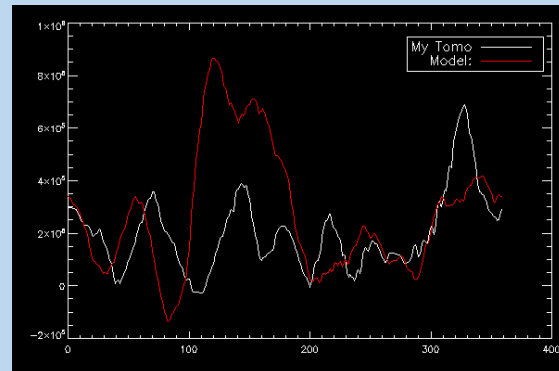
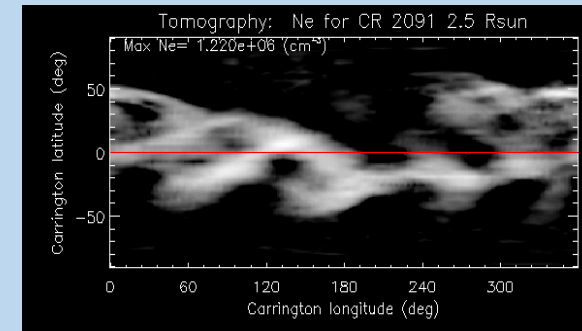
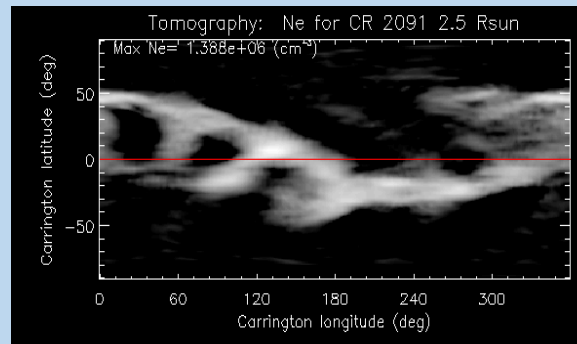
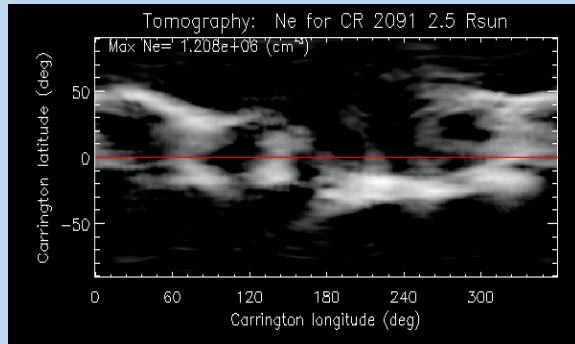
- **Uncertainties of 3D density due to time evolution**

$N=64^3$, $w=1/\sqrt{N_{bg}(r)}$
 Period: - 7 days

$N=64^3$, $w=1/\sqrt{N_{bg}(r)}$
 Period: Jun-23 to Jul-07, 2010

$N=64^3$, $w=1/\sqrt{N_{bg}(r)}$
 Period: + 7days

(zero-order regularization with radial weighting)

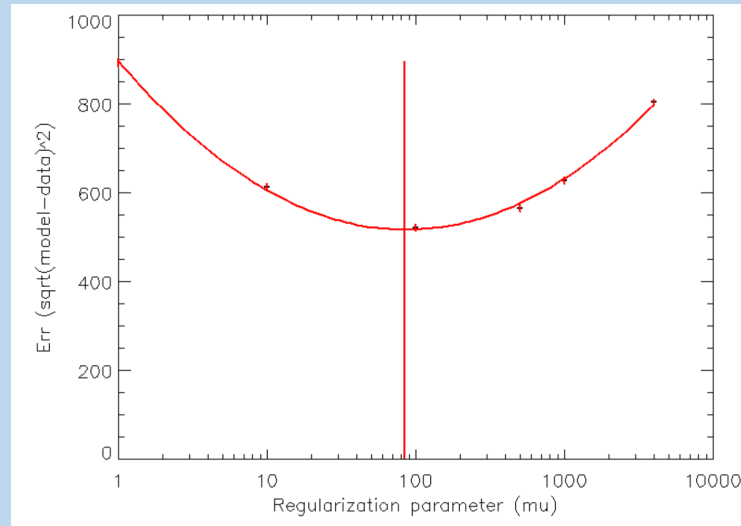
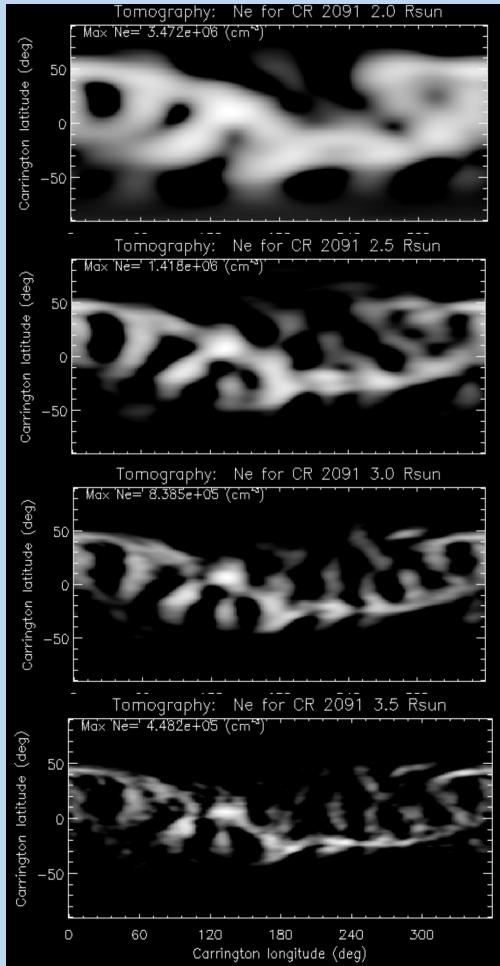


Red: analyzed case
 White: - 7 days

Red: analyzed case
 White: + 7 days

■ Cross-Validation Method

to determine the best regularization parameter and provide the error estimate

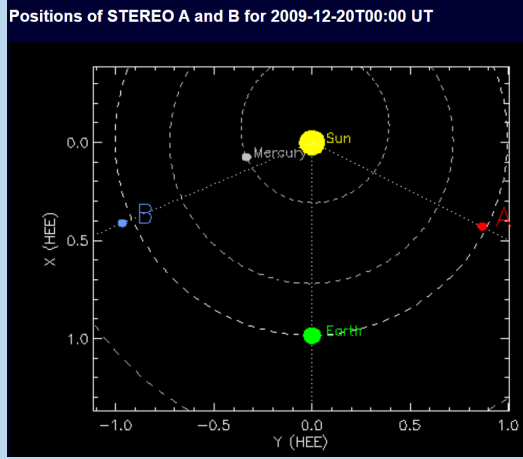


- 5-fold cross-validations calculated based on randomly sampling rate of 10% for CR 2098 (grid=128³ without radial weighting)

- 1) Calculate the solution from data with 10-20% extracted randomly
- 2) Validate the solution by calculating the error between model-prediction and the extracted data
- 3) Repeat steps 1-2 for 5 times for different μ values to determine μ_{best} with the minimum Err
- 4) Standard deviation for average of 5-fold solutions with μ_{best} giving error estimates of the solution

- Best regularization-parameter, $\mu=85$, determined from 5-fold CVs

Reconstruction by tomography from observations of multiple spacecraft

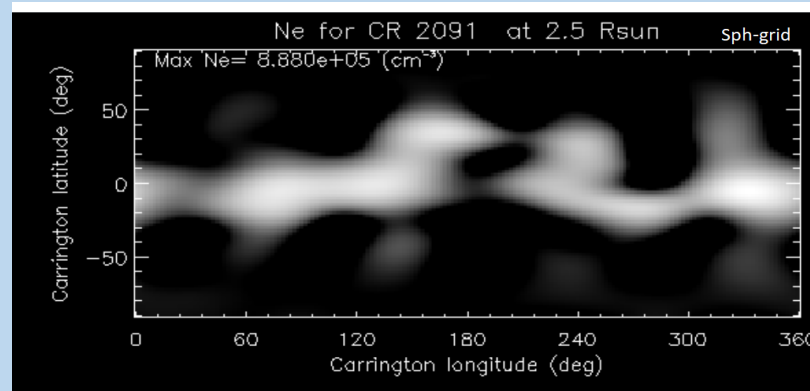


On 2009/12/20

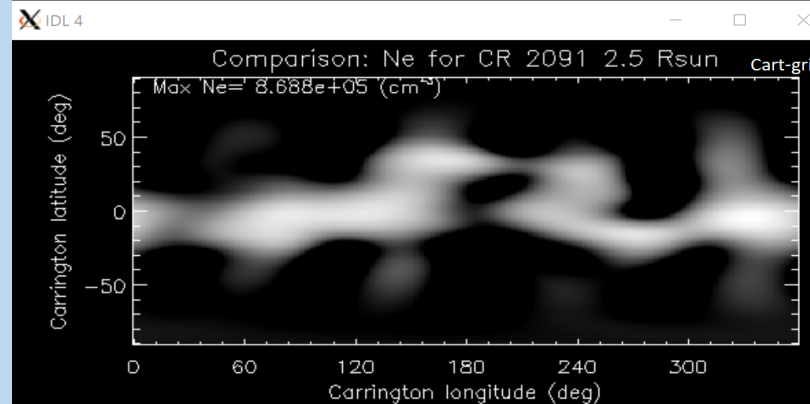
$$\phi_{AE} = 64^\circ$$

$$\phi_{BE} = 67^\circ$$

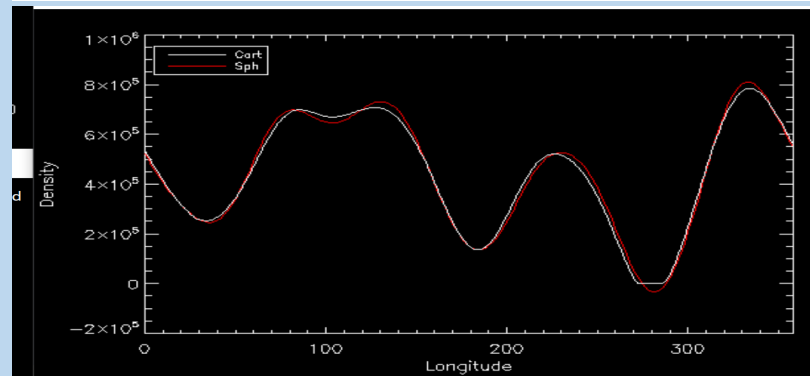
➤ CR 2091 in Dec 2009 during minimum of SC 24



Cartesian-F90
grid=128³, wt=1,
mu=1000



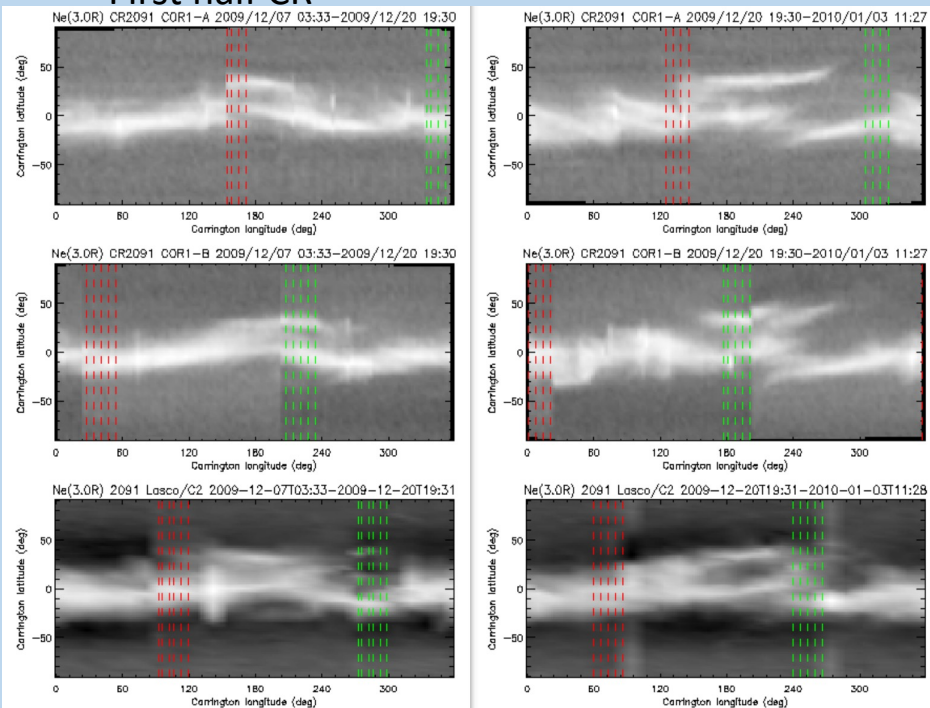
Sphere-F90
grid=p181t91r37,
wt=1, mu=100



Comparison of
density profiles
along the equator
between Cartesian
and spheric grids

First half CR

Second half CR

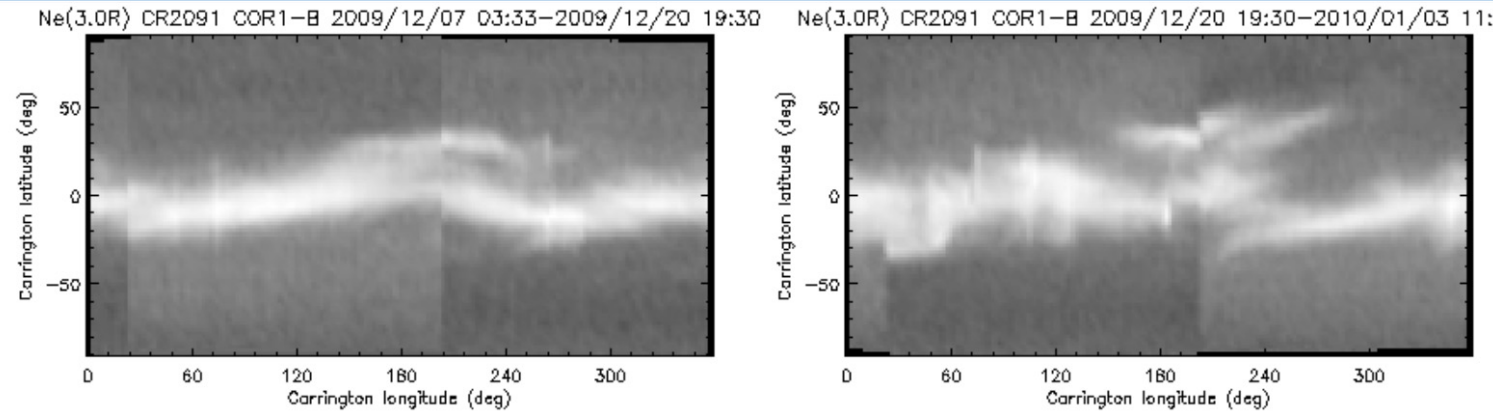


E-limb (red) and W-limb (green) positions of sampling data from COR1-A (upper), COR1-B (middle), LASCO/C2 (bottom) - **data coverage time: ~ 5 days**

- Comparison between single-vantage and 3 vantage reconstructions for CR2091

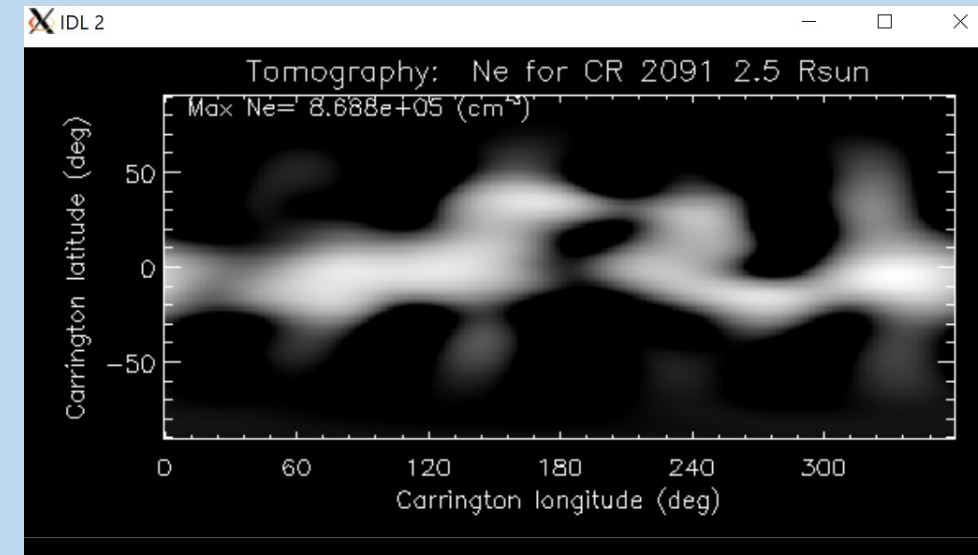
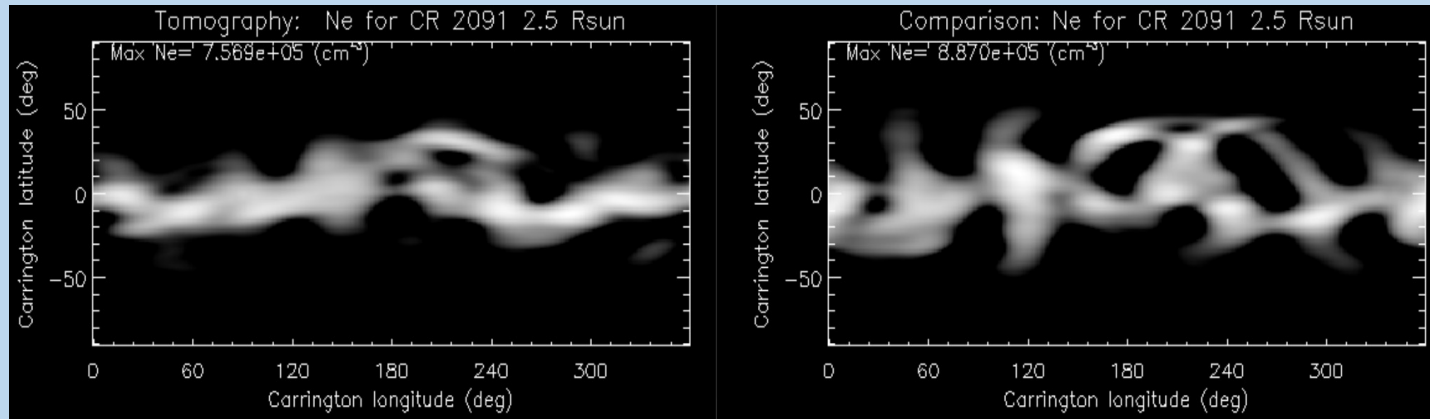
First half CR

Second half CR



Tomographic reconstruction from a 5-days observations near central CR using 3 views from COR1-A, COR1-B, and Lasco-C2

Synoptic density reconstruction formed SSPA method from COR1-B



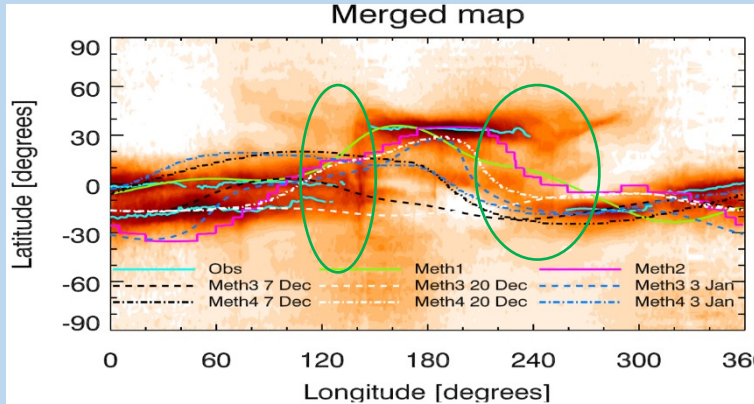
Cartesian-F90 grid=128³, wt=1, mu=1000

Tomographic density reconstructions from pB images of COR1-B

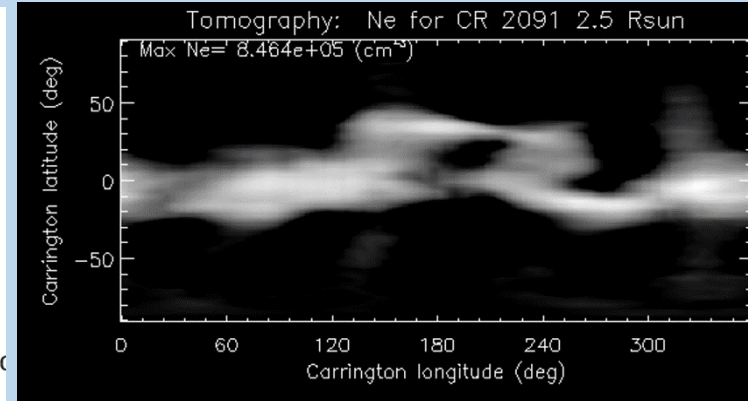
- Some Applications of the tomographic 3D coronal density

- Evaluate coronal magnetic field model by comparing predicted current Sheet (red line) with locations of density peaks

CR 2091



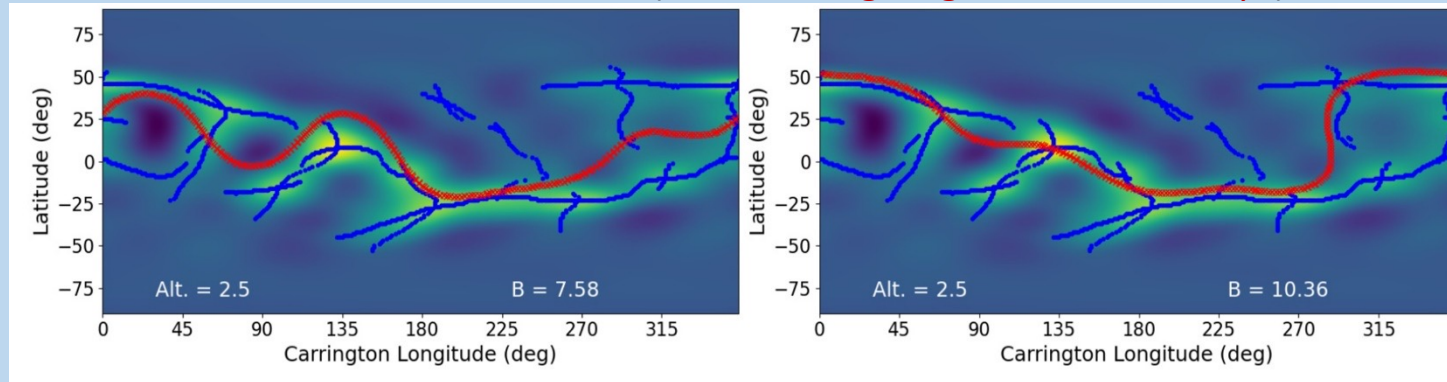
pB synoptic map merged from LASCO/C2, STEREO/COR1-A and B data (from Sasso et al. 2019)



Tomographic density reconstructed from the same pB observations

Tomographic density can overcome the defects when using the pB synoptic map as agent

Backbone metric (Jones, Wang, Arge, et al., 2022, ApJ)



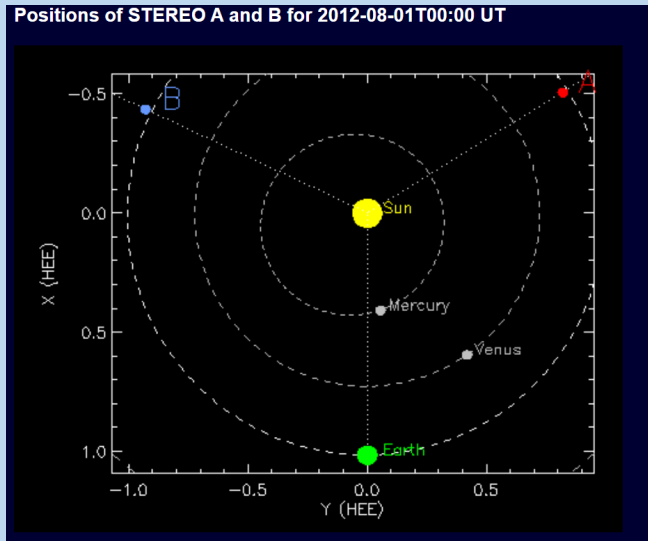
Model-1: data times close to tomo.

Model-2: with farside-emerged AR

$$P(\chi_i) = e^{\kappa \cos(\min(\theta_i))},$$

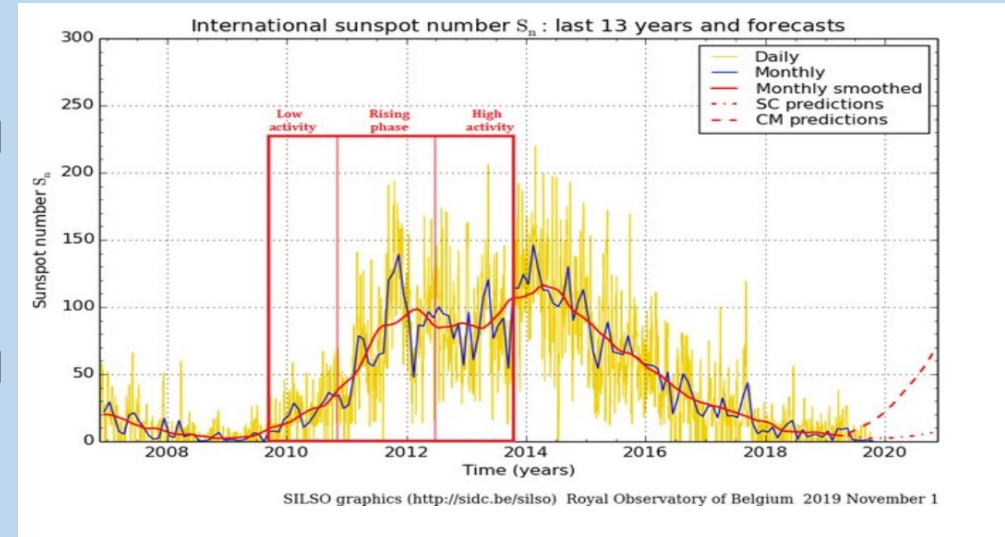
$$M = \frac{\overline{P(\chi_{NL})}}{\overline{P(\chi_{shell})}}.$$

- Reconstructions of CR 2123-2126 during maximum of Solar Cycle 24 (2012/04-08)

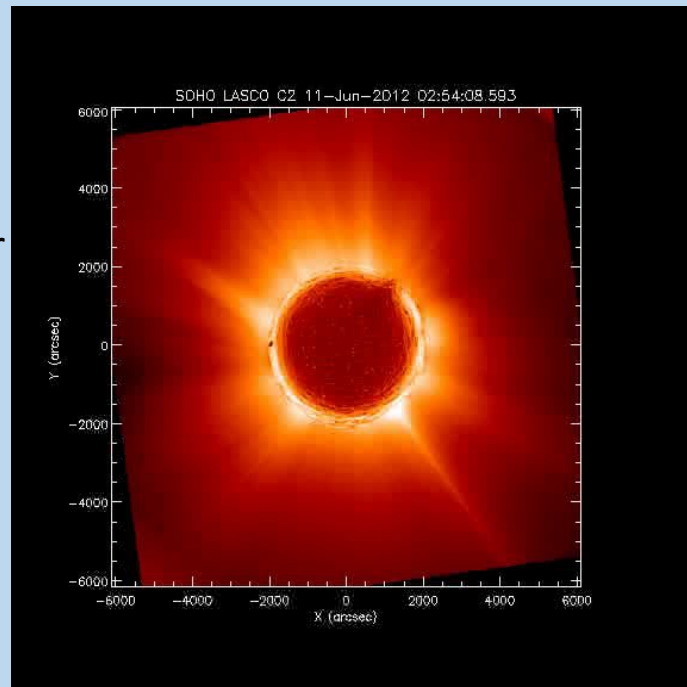


Separation between COR1-A and Earth: $\phi_{AE} = 122$ deg (58)
 Separation between COR1-B and Earth: $\phi_{BE} = 115$ deg (65)
 Separation between COR1-A and COR1-B: $\phi_{AB} = 123$ deg (57)

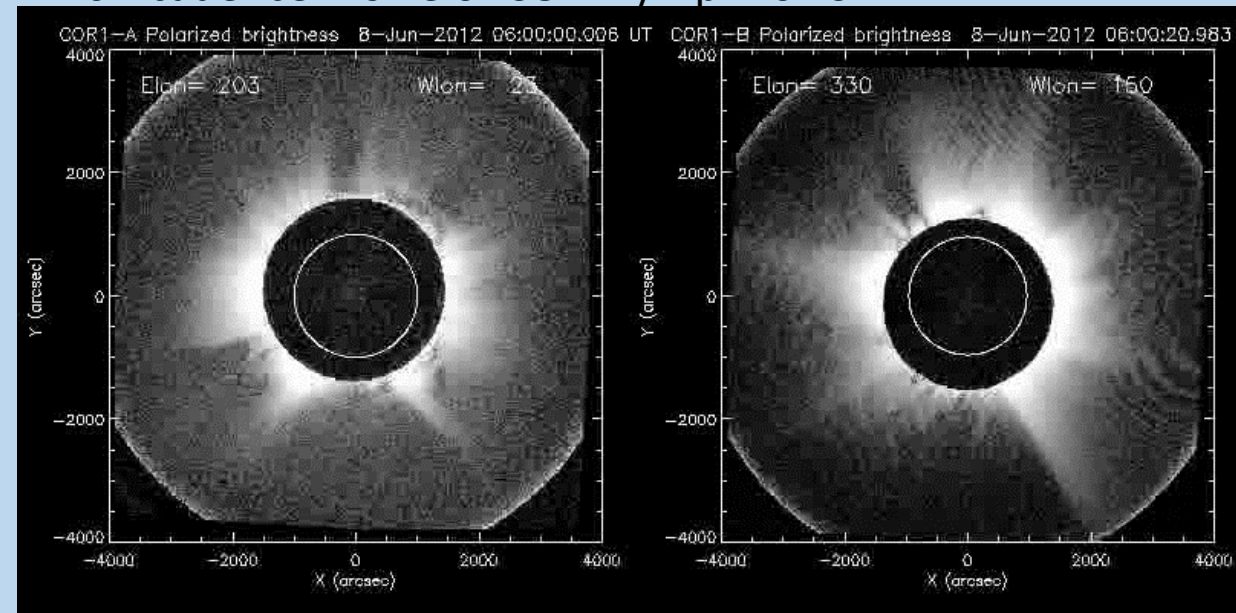
Bigazzi+2020 AG



3-5 pB images/day for LASCO/C2 FOV 2-6 Rs



6h-cadence movie of COR1A/B pB for CR 2124



Reconstructions of CR 2123-2126 during maximum of Solar Cycle 24 (2012/04-08)

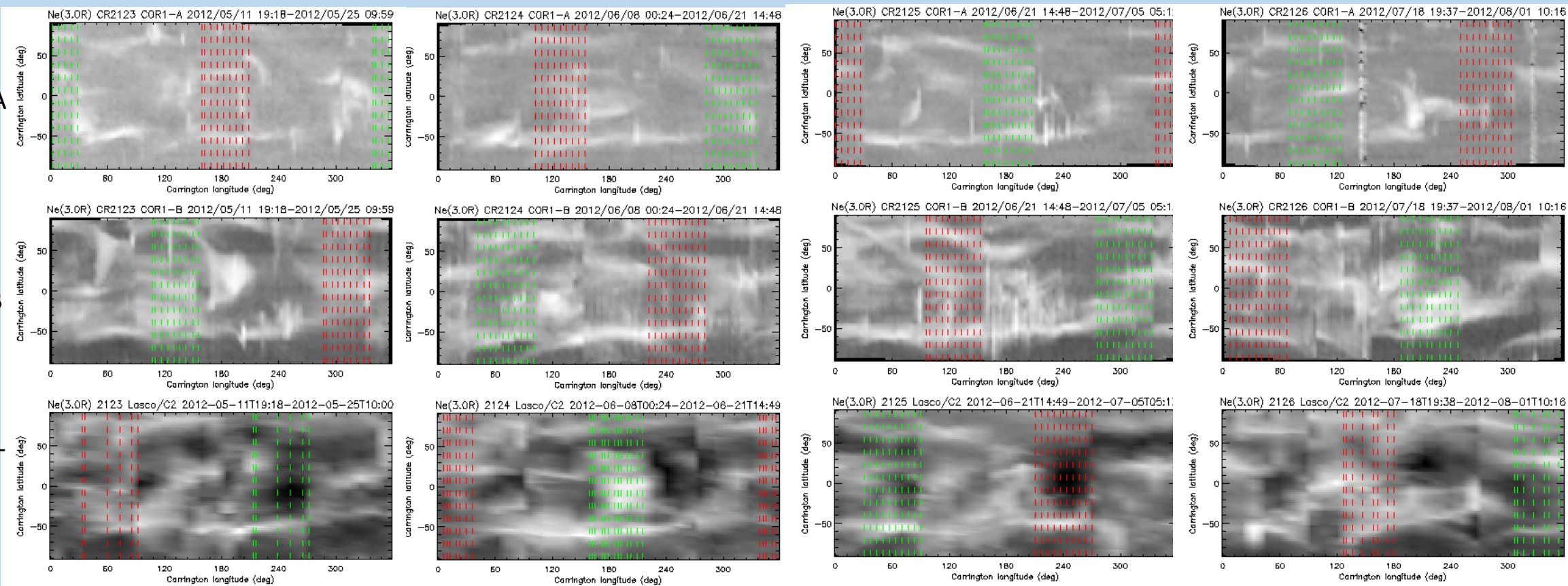
Data selections:

- Avoid CMEs (esp. long-duration eruption; streamer blowout)
- Avoid images with strong inference strips

COR1-A

COR1-B

LASCO C2



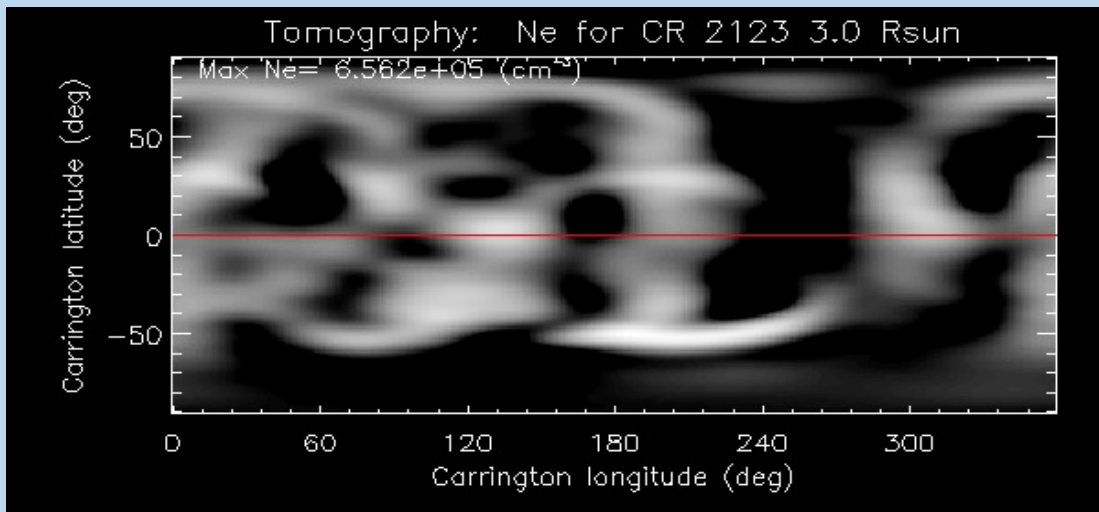
CR 2123

CR 2124

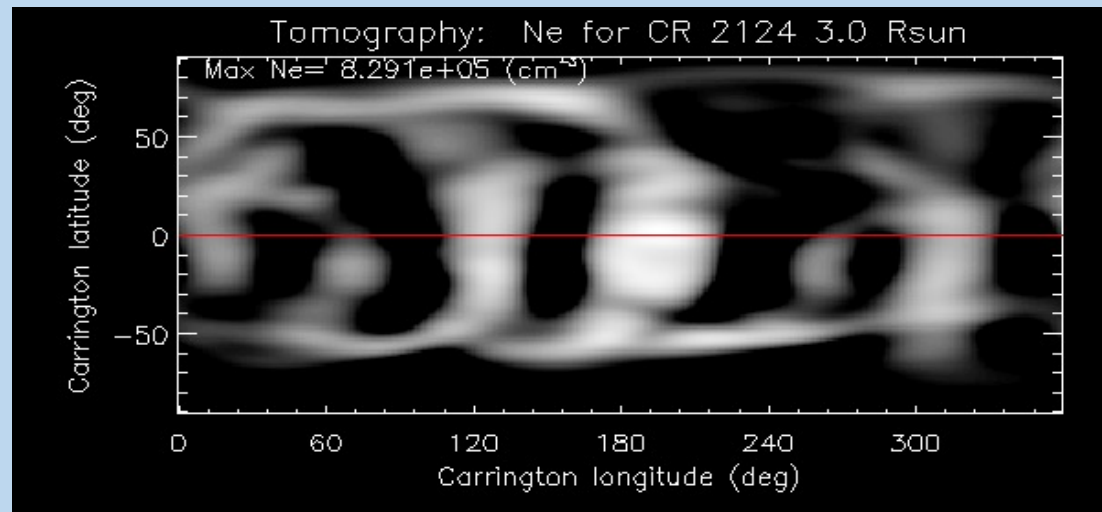
CR 2125

CR 2126

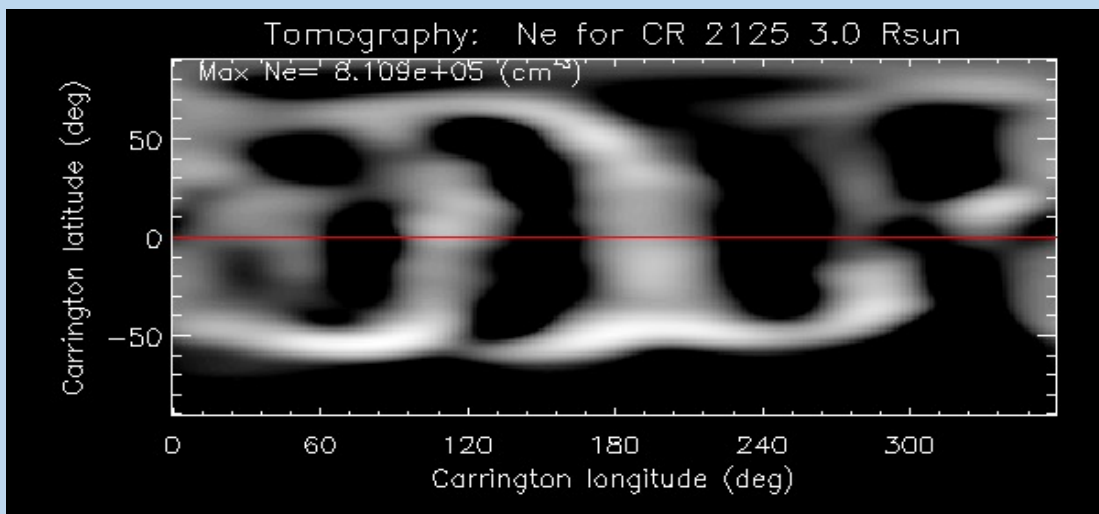
Cartesian-F90, 3-views, grid=128³, wt=1, $\mu=100$, reg=d2f2



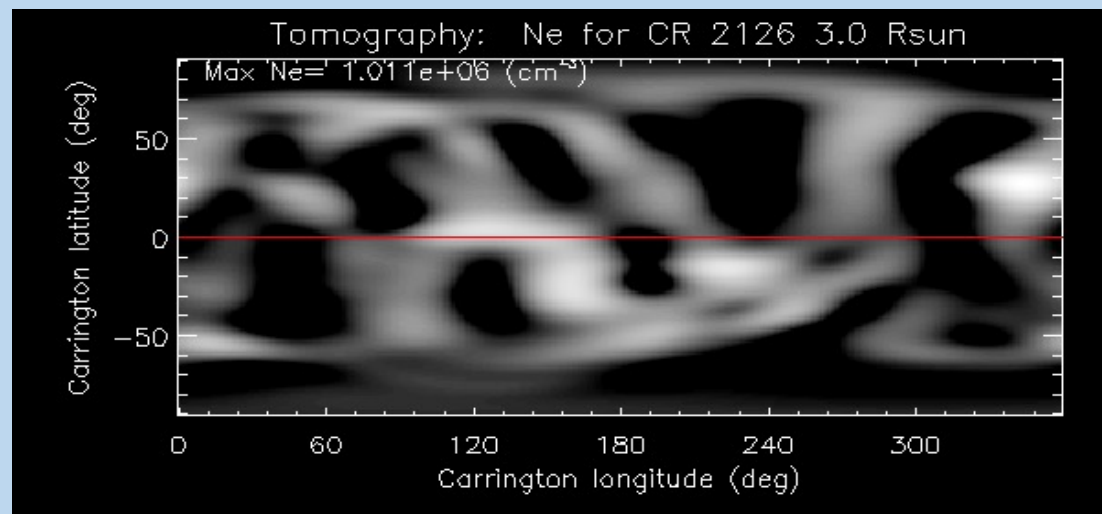
2012/05/11 12:00 – 2012/05/16 03:00 for CR 2123



2012/06/12 00:05 – 2012/06/16 12:00 for CR 2124

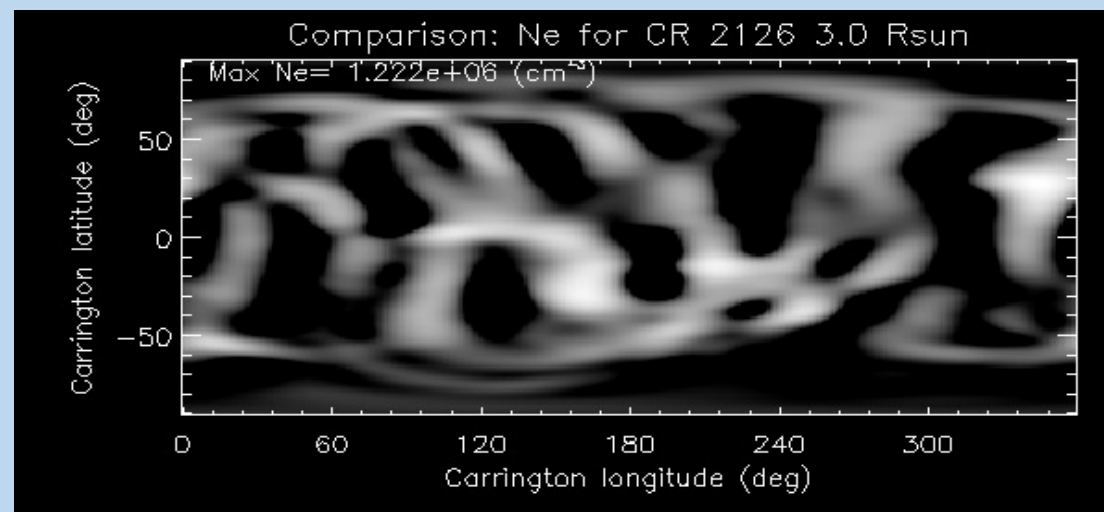
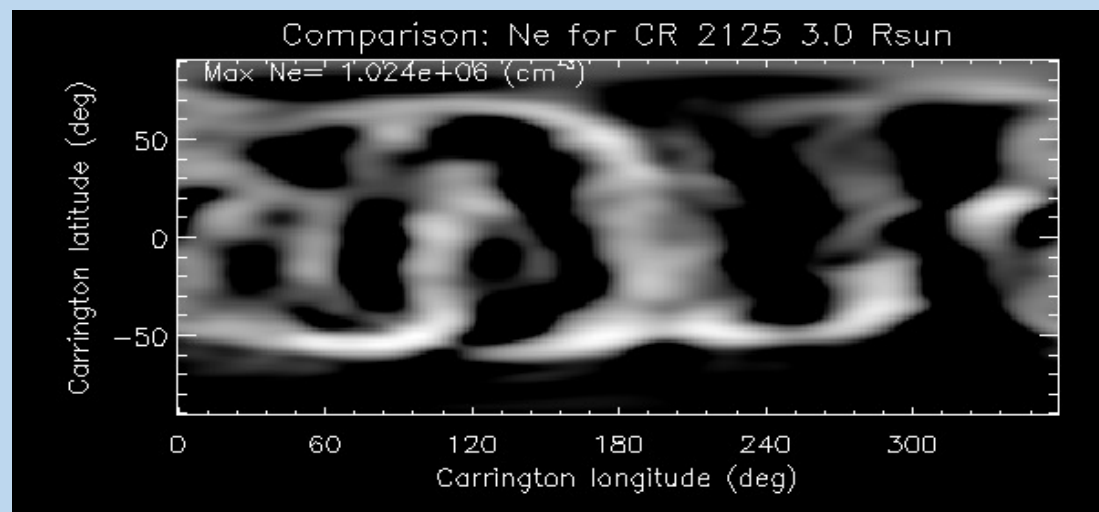
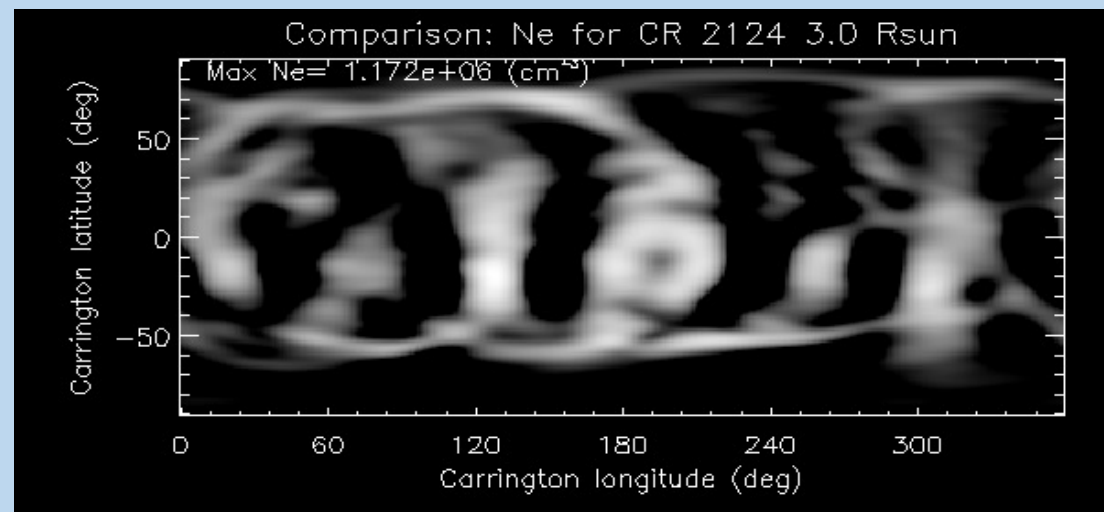
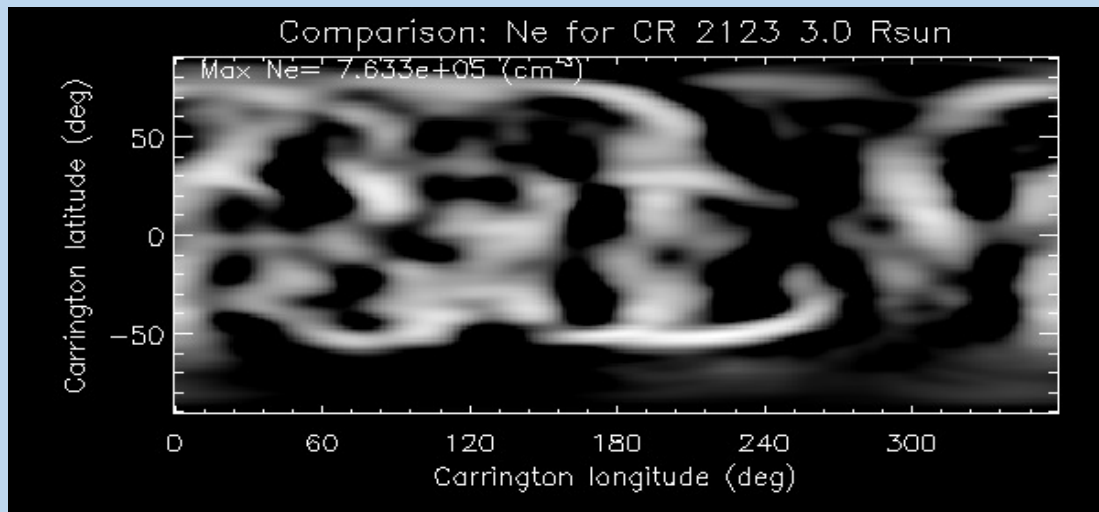


2012/06/21 15:00 – 2012/06/26 03:00 for CR 2125

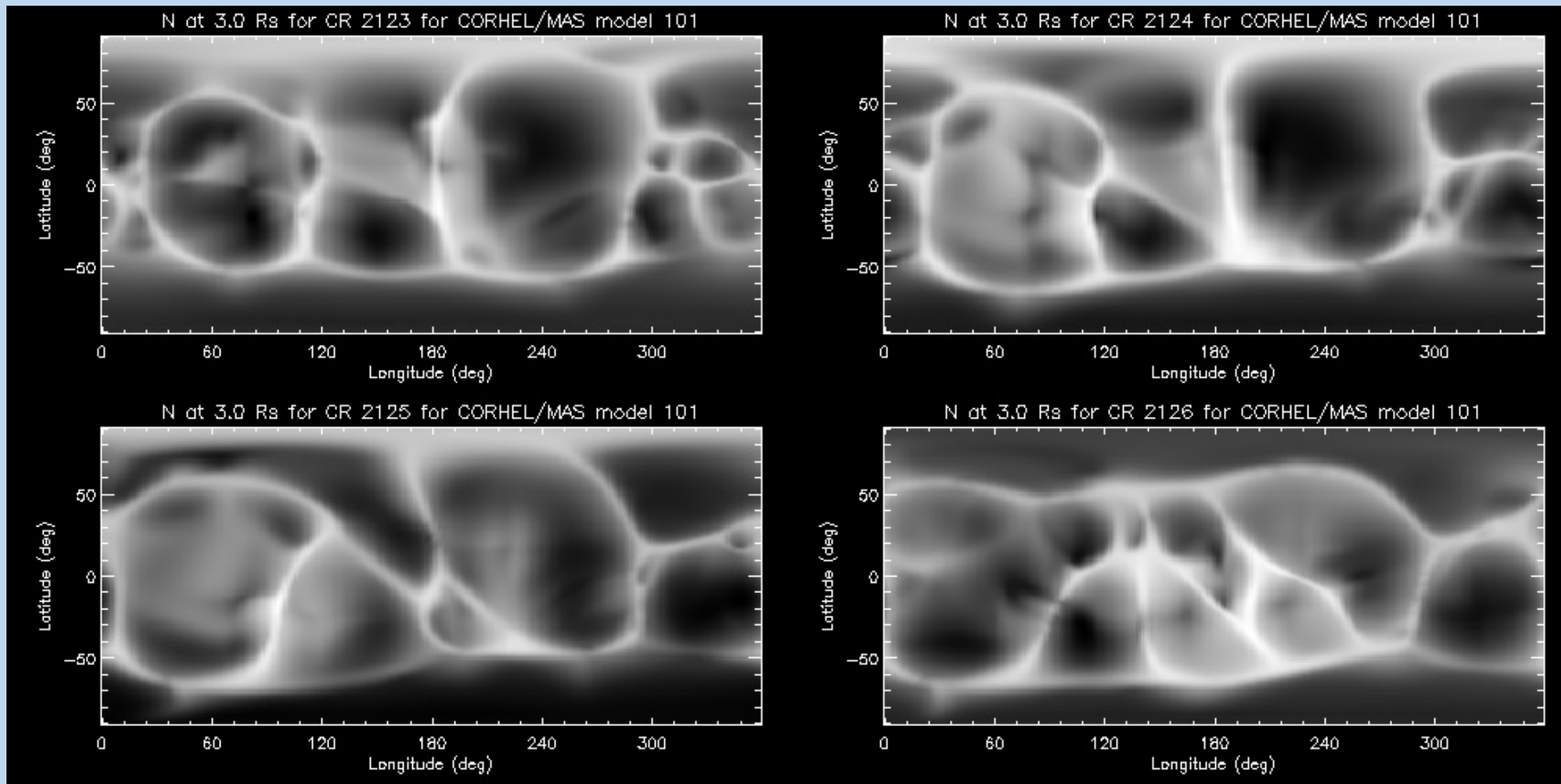


2012/07/25 12:00 – 2012/07/30 00:05 for CR 2126

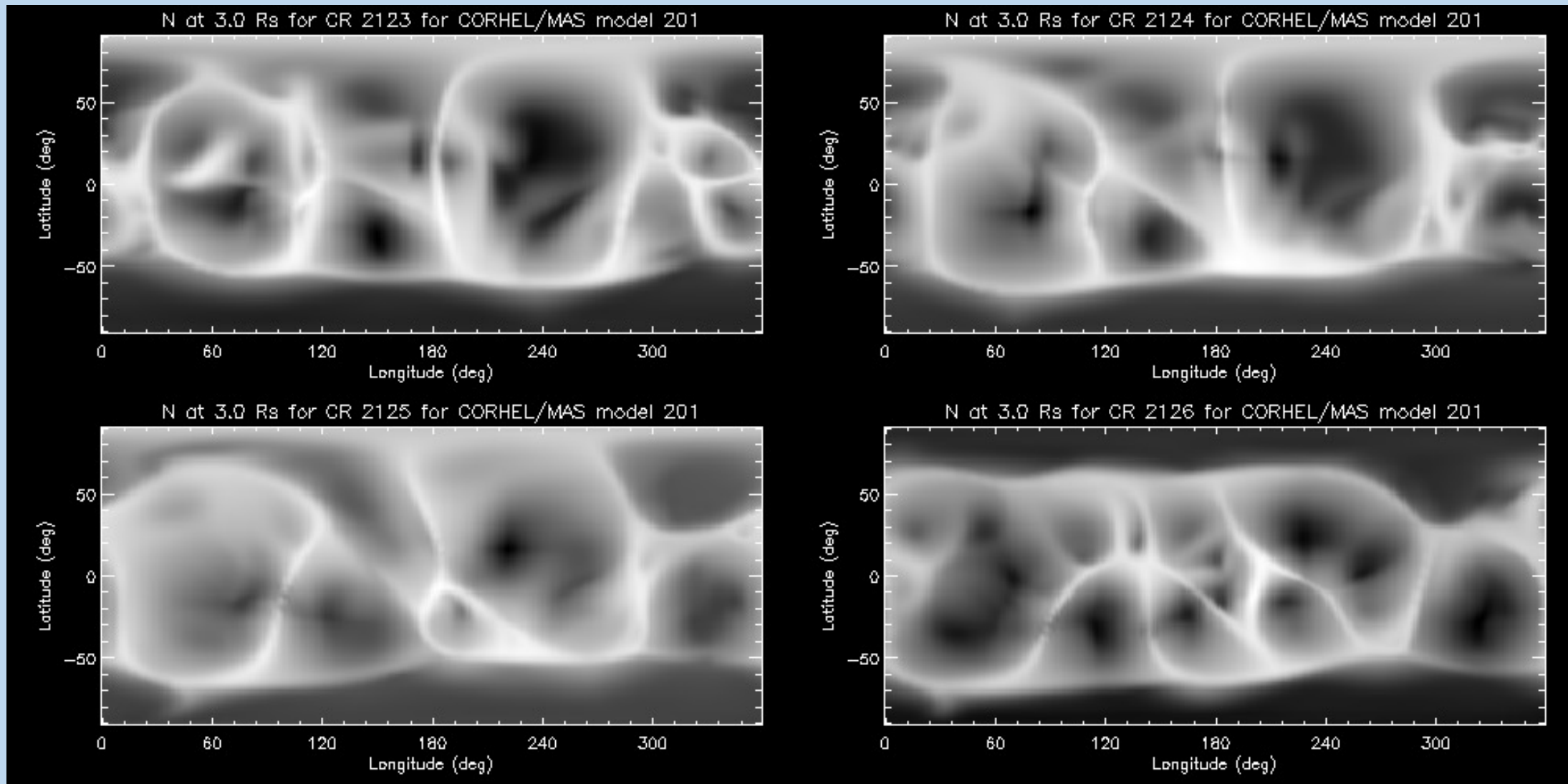
Cartesian-F90, 3-views, grid=128^3, wt=1, $\mu=10$, reg=d2f2



Thermodynamic MHD steady-state solution with heating model 101 from PSI

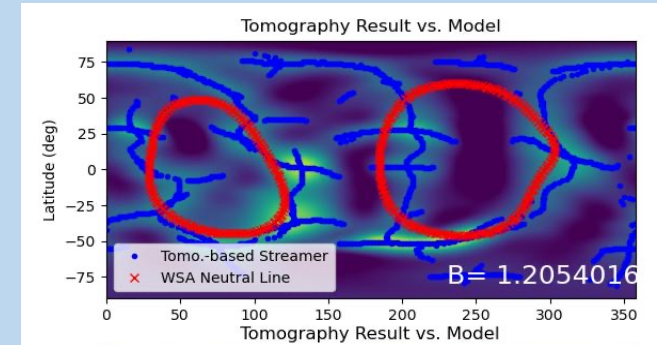
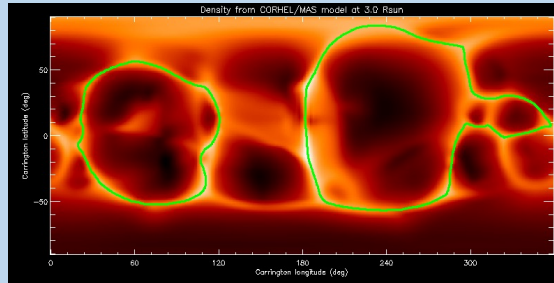
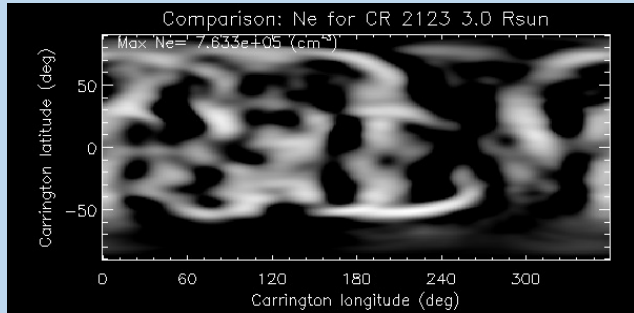


Thermodynamic MHD steady-state solution with heating model 201 from PSI

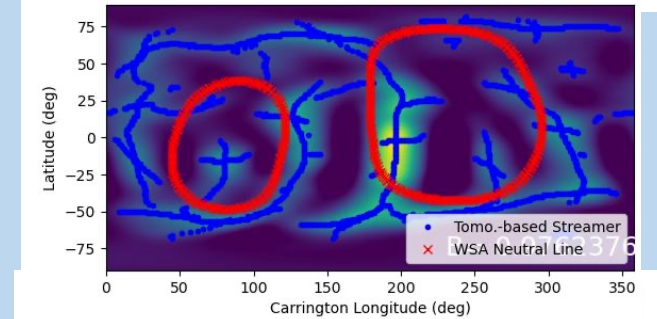
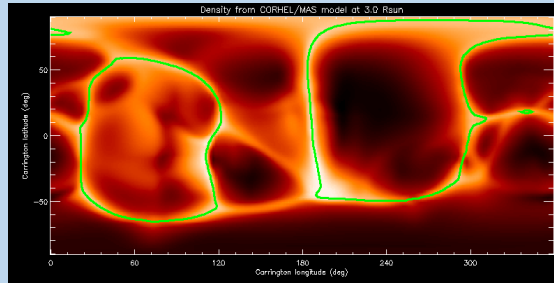
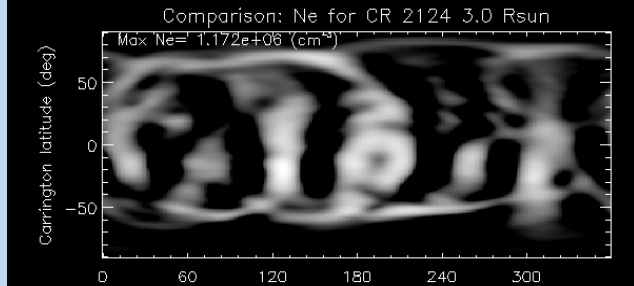


■ Comparison of Density at $r=3.0 R_s$ between tomography and MHD model

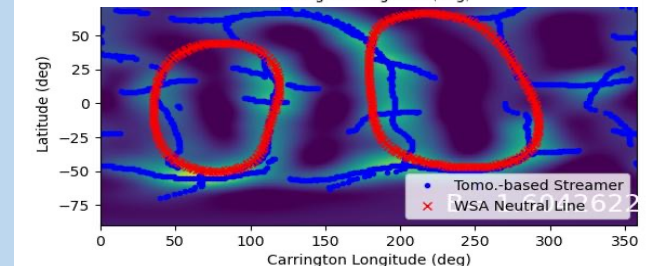
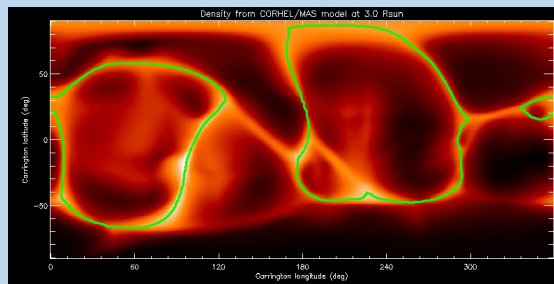
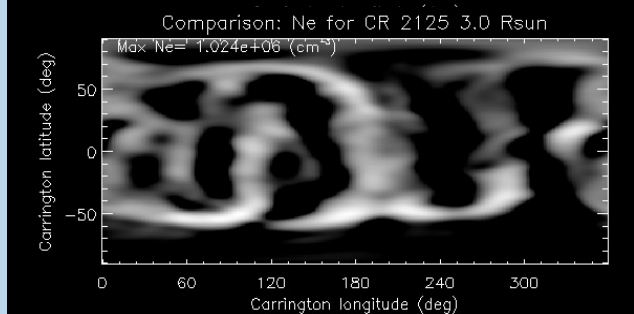
CR 2123



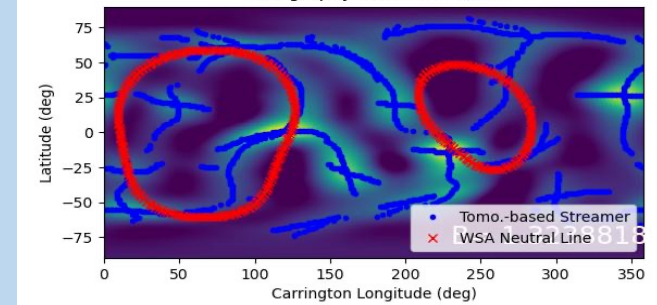
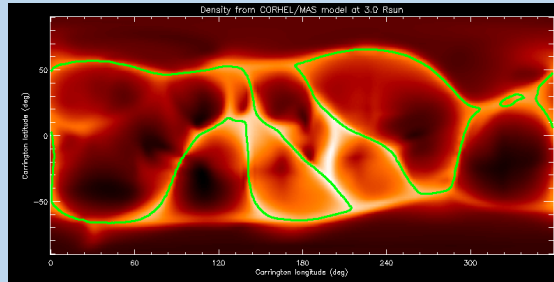
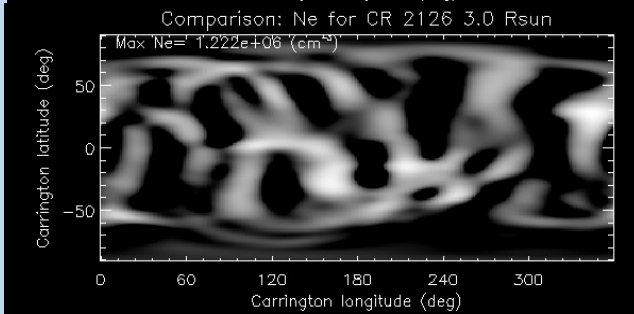
CR 2124



CR 2125



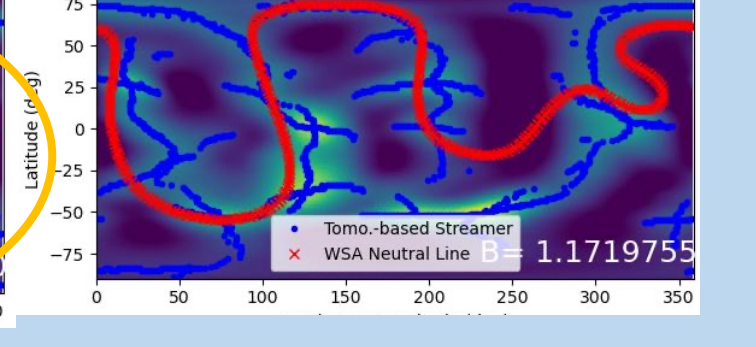
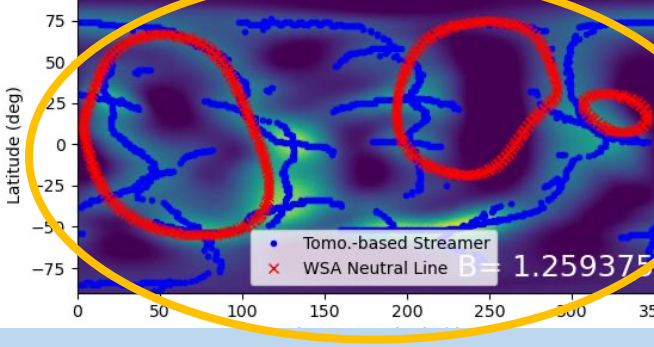
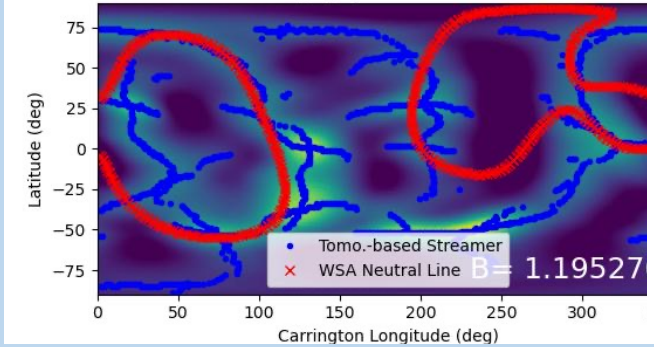
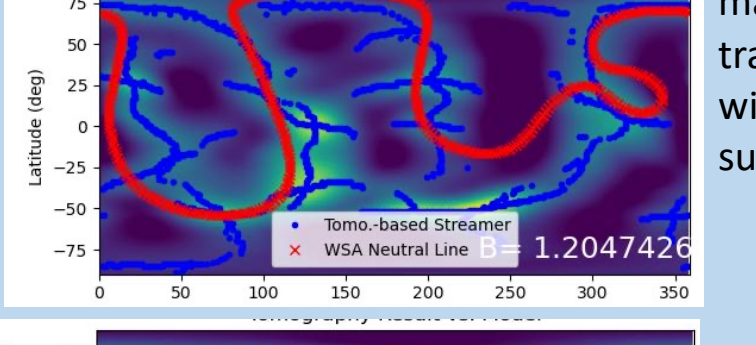
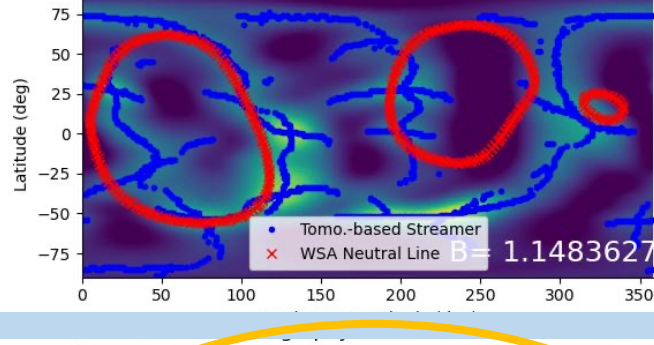
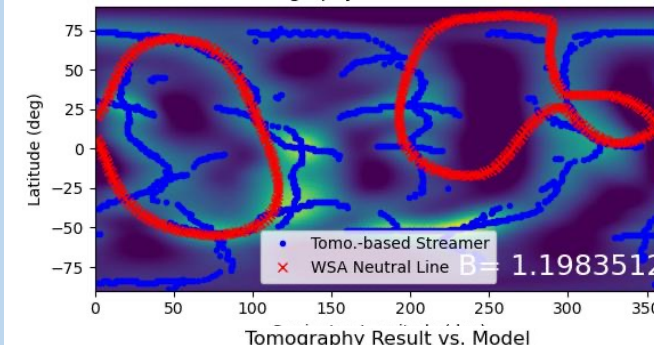
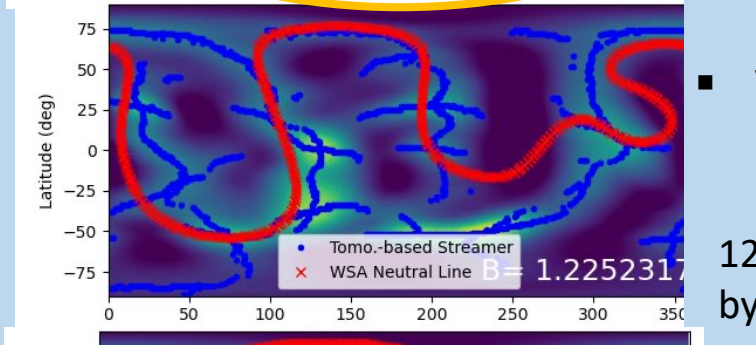
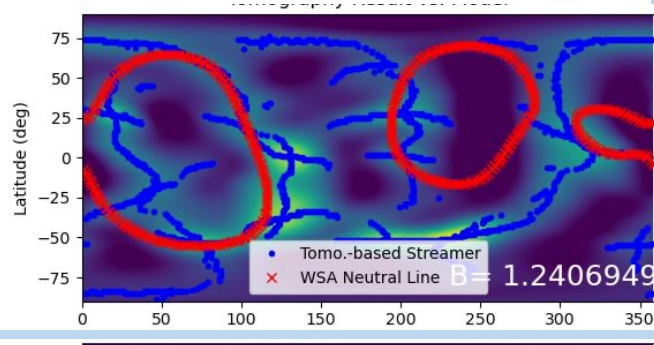
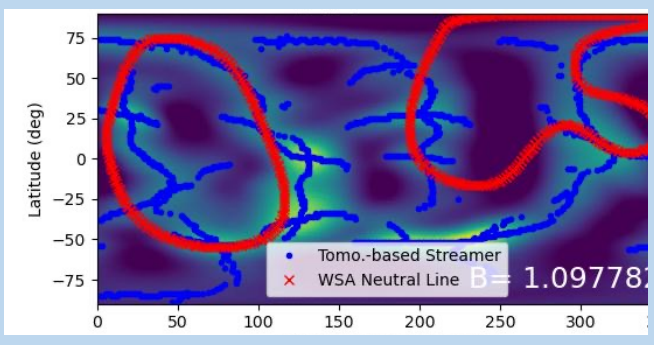
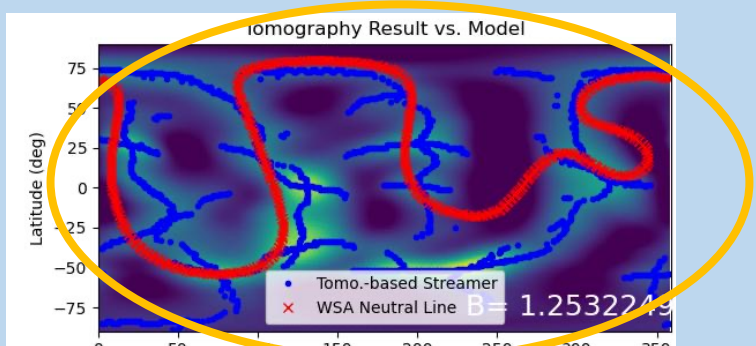
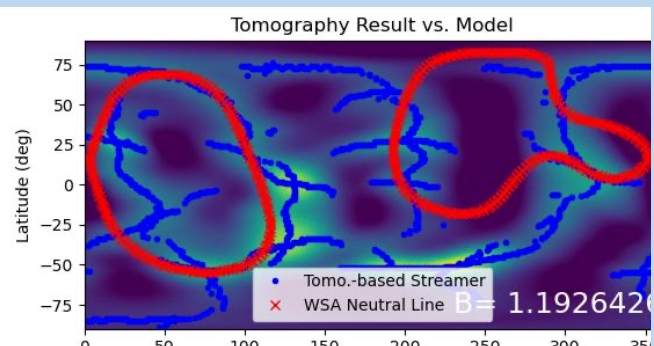
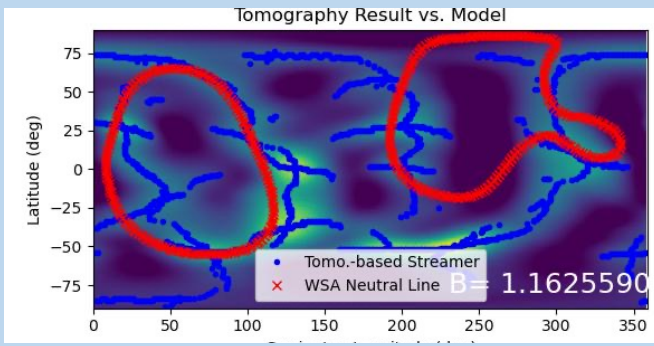
CR 2126



Tomography N from 3 views

PSI MHD steady-state N at $r=3 R_s$
(green contours for $B_r=0$)

Tomography backbone metric vs.
WSA model with Synoptic boundary



Validation of WSA model prediction

12 realizations produced by ADAPT-GONG input maps, where a flux transport model was used with the variations due to supergranular motion

- **Perspectives:** tomography reconstructions from multiple vantage coronagraph pB observations of existing and future missions

➤ Plans:

- Reconstruction of 3D density from STEREO/COR1-A/B and LASCO/C2 during solar minimum and maximum of Solar Cycle 24 for 2.2-4.0 Rs
- Reconstruction from STEREO/COR2 and LASCO/C3 for large FOV 6-15 Rs (e.g., In 2009/07/01-2010/08/01, solar minimum to rising phase of SC24): Separation angle $\phi_{A/B-E} \sim 50^\circ - 75^\circ$, maximum required $P_{\text{obs}} \leq 6$ days
- Reconstruction from STEREO/COR2A (2-15 Rs), LASCO/C3 (6-32 Rs), SoHO/METIS (1.7-9 Rs), PUNCH/NFI (6-32 Rs), and CODEX (3-10 Rs)

➤ Main issues

- Corrections for F-corona
- Cross calibrations
- Effects of coronal dynamic, eruptions, and evolution

Summary

- Develop the regularized tomography code in F90 on both Cartesian and Spheric grids
- Improvements with radial weighting and zero-order regularization to alleviate smoothing effects and use cross-validation to estimate errors
- Demonstrate reconstructions of 3D electron density using pB images from STEREO/COR1-A, COR1-B, and LASCO/C2 during the solar minimum and maximum of SCs
- Reconstructions on a timescale of 4-5 days data enable the studies of coronal structure evolution providing crucial constraints on solar wind models